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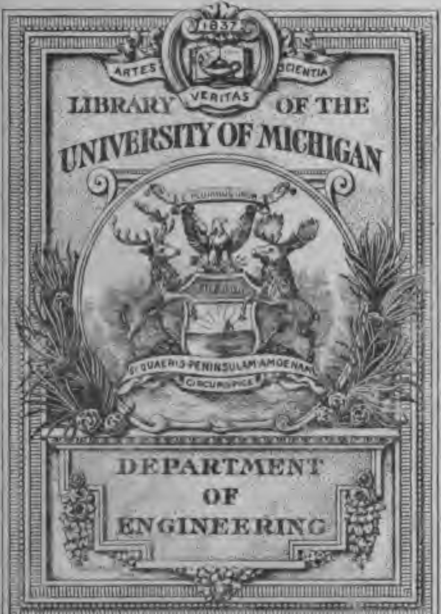

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DYNAMIC ELECTRICITY,

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Its Modern Use and Measurement, chiefly in its
Application to Electric Lighting and Telegraphy;

INCLUDING

I. SOME POINTS IN ELECTRIC LIGHTING,

By Dr. John Hopkinson, F. R. S.

II. On the Measurement of Electricity for Commercial Purposes,

By James N. Shoolbred, M. Inst. C. E.

III. ELECTRIC LIGHT ARITHMETIC,

By R. E. Day, M. E.

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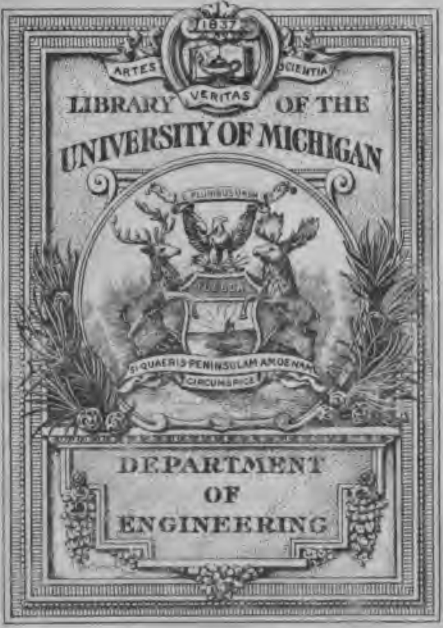

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I.

SOME POINTS

—IN—

ELECTRIC LIGHTING.

BY

DR. JOHN HOPKINSON,

F. R. S., M. Inst. C. E.

Release 4-28-42 mgl

PREFACE.

THE component parts of this little book relate entirely to the practical uses of electricity.

Dr. Hopkinson's lecture before the Institution of Civil Engineers presents the subject of Electric Lightning in the aspect of an engineering problem, in which the *quantities* in question are as satisfactorily estimated as in hydraulics.

Mr. Shoolbred's paper, prepared for the Society of Telegraph Engineers, is an extension of the subject in the same direction. As it was prepared for practical electricians it is the more technical of the two papers.

The problems are selected from R. E. Day's excellent compilation known as Electric Light Arithmetic.

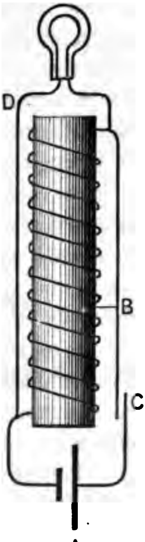
SOME POINTS IN ELECTRIC LIGHTING.

Artificial light is generally produced by raising some body to a high temperature. If the temperature of a body be greater than that of surrounding bodies it parts with some of its energy in the form of radiation. Whilst the temperature is low these radiations are not of a kind to which the eye is sensitive; they are exclusively radiations less refrangible and of greater wave-length than red light, and may be called infra red. As the temperature is increased the infra red radiations increase, but presently there are added radiations which the eye perceives as red light. As the temperature is further increased, the red light increases, and yellow, green and blue rays are successively thrown off in addition. On pushing the temperature to a still higher point, radiations of a wave-length, shorter even than violet light, are produced, to which the eye is insensitive, but which act strongly on certain chemical substances; these may be called ultra violet rays. It is thus seen that a very hot body in general throws out rays of various wave-lengths, our eyes, it so happens, being only sensitive to certain of these, viz., those not very long and not very short, and that the hotter the body the more of every kind of radiation will it throw out; but the proportion of short waves to long waves becomes vastly greater as the temperature is increased. The problem of the artificial production of light with economy of energy is the same as that of raising some body to such a temperature that it shall give as large a proportion as possible of those rays which the eye happens to be capable of feeling. For practical purposes this temperature is the highest temperature we can produce. Owing to the high temperature at which it remains solid, and to its great emissive power the radiant body used for artificial illumination is nearly always some form of carbon. In the electric current we have an agent whereby we can convert more energy of other forms into heat in a small space than in any other way; and fortunately carbon is a conductor of electricity as well as a very refractory substance.

The science of lighting by electricity very naturally divides itself into two principal parts—the methods of production of electric currents, and of conversion of the energy of those currents into heat at such a temperature as to be given off in radiations to which our eyes are sensible. There are other subordinate branches of the subject, such as the consideration of the conductors through which the electric energy is transmitted, and the measurement of the quantity of electricity passing and its potential or electric pressure. Although I shall have a word or two to say on the other branches of the subject, I propose to occupy most of the time at my disposal this evening with certain points concerning the conversion of mechanical energy into electrical energy. We know nothing as to what electricity is, and its appeals to our senses are in general less direct than those of the mechanical phenomena of matter. The laws, however, which we know to connect together those phenomena which we call electrical, are essentially mechanical in form, are closely correlated with mechanical laws, and may be most aptly illustrated by mechanical analogues. For example, the terms "potential," "current," and "resistance," with which we are becoming familiar in electricity have close analogues respectively in "head," "rate of flow," and "coefficient of friction" in the hydraulic transmission of power. Exactly as in hydraulics, head multiplied by velocity of flow is power measured in foot-pounds per second or in horse-power, so potential multiplied by current is power and is measurable in the same units. The horse-power not being a convenient electrical unit, Dr. Siemens has suggested that the electrical unit of power or volt-ampere should be called a watt: 746 watts are equal to one horse-power.

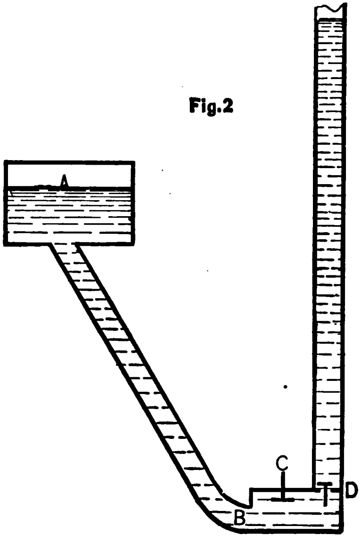
Again, just as water flowing in a pipe has inertia and requires an expenditure of work to set it in motion, and is capable of producing disruptive effects if its motion is too suddenly arrested—as, for example, when a plug-tap is suddenly closed in a pipe through which water is flowing rapidly, so a current of electricity in a wire has inertia; to set it moving electro-motive force must work for a finite time, and if we attempt to arrest it suddenly by breaking the circuit, the electricity forces its way across the interval as a spark. Corresponding to mass and moments of inertia in mechanics we have in electricity coefficients of self-induction. We will now show that an electric circuit behaves as though it had inertia. The apparatus we shall use is shown diagrammatically in Fig. 1. A current from a Sellon battery A circulates round an electro-magnet B: it can be made and broken at pleasure at C. Connected to the two extremities of the wire on the magnet is a small incandescent lamp D, lent me by Mr. Crompton, of many times

Fig. 1



the resistance of the coil. On breaking the circuit, the current in the coil, in virtue of its momentum, forces its way through the lamp, and renders it momentarily incandescent, although all connection with the battery, which in any case would be too feeble to send sufficient current through the lamp, has ceased. Let us try the experiment, make contact, break contact. You observe the lamp lights up. Compare with the diagram Fig. 2 of the hydraulic analogue the hydraulic ram. There a current of water suddenly arrested forces a way for a por-

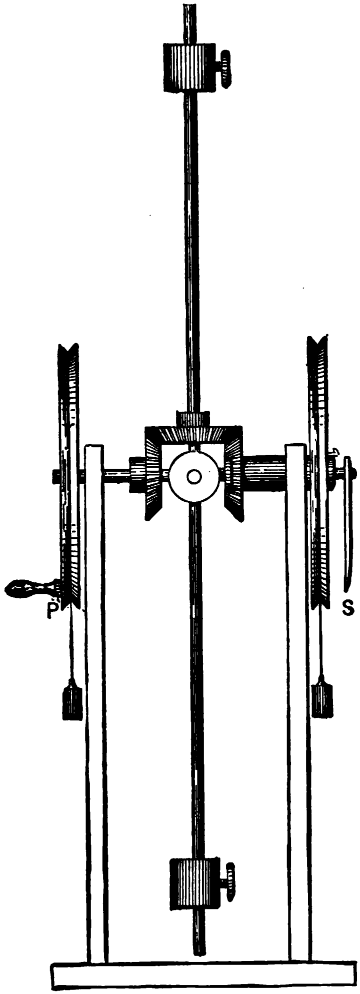
Fig.2



tion of its quantity to a greater height than that from which it fell. A B corresponds to the electro magnet, the valve C to the contact-breaker, and D E to the lamp. There is, however, this difference between the inertia of water in a pipe and the inertia of an electric current—the inertia of the water is confined to the water, whereas the inertia of the electric current resides in the surrounding medium. Hence arise the phenomena of induction of currents upon currents, and of magnets upon moving conductors—phenomena which have no immediate analogues in hydraulics. There is thus little difficulty to any one accustomed to the laws of rational mechanics in adapting the expression of these laws to fit electrical phenomena; indeed we may go so far as to say that the part of electrical science with which we have to deal this evening is essentially a branch of mechanics, and as such I shall endeavor to treat it.

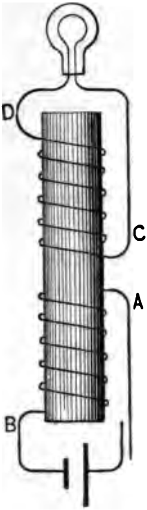
This is neither the time nor the place for setting forth the fundamental laws of electricity, but I cannot forbear from showing you a mechanical illustration, or set of mechanical illustrations, of the laws of electrical induction, first discovered by Faraday. I have here a model, Fig. 3, which was made to the instructions of the late Professor Clerk Maxwell, to illustrate the laws of induction. It consists of a pulley P, which I now turn with my hand, and which represents one electric circuit, its motion the current therein. Here is a second pulley S, representing a second electric circuit. These two pulleys are geared together by a simple differential train, such as is sometimes used for a dynamometer: The intermediate wheel of the train, however, is attached to a balanced fly-wheel, the moment of inertia of which can be varied by moving inwards or outwards these four brass weights. These resistances of the two electric circuits are represented by the friction on the pulleys of two strings, the tension of which can be varied by tightening these elastic bands. The differential train, with its fly-wheel, repre-

Fig. 3



sents the medium, whatever it may be, between the two electric conductors. The mechanical properties of this model are of course obvious enough. Although the mathematical equations which represent the relation between one electric conductor and another, in its neighborhood, are the same in form as the mathematical equations which represent the mechanical connection between these two pulleys, it must not be assumed that the magnetic mechanism is completely represented by the model. We shall now see how the model illustrates the action of one electric circuit upon another. You know that Faraday discovered that if you have two closed conductors arranged near to and parallel to each other, and if you cause a current of electricity to begin to flow in the first, there will arise a temporary current in the opposite direction in the second. This pulley, marked P on the diagram, represents the primary circuit, and the pulley marked S on the diagram the secondary circuit. We cause a current to begin to flow in the primary, or turn the pulley P; an opposite current is induced in the secondary circuit, or the pulley S turns in the opposite direction to that in which we began to move the pulley P. The effect is only temporary, resistance speedily stops the current in the secondary circuit, or in the mechanical model friction the rotation of the pulley S. I now gradually stop the motion of P; the pulley S moves in the direction in which P was previously moving, just as Faraday found that the cessation of the primary current induced in the secondary circuit a current in the same direction as that which had existed in the primary. If there were a large number of convolutions or coils in the secondary circuit, but that circuit were not completed, but had an air space interrupting its continuity, an experiment with the well-known Ruhmkorff coil would show you that when the current was *suddenly* made to cease to flow in the primary circuit, so great an electromotive force would be exerted in the secondary circuit that the electricity would leap across the space as a spark. I will now show you what corresponds to a spark with this mechanical model. The secondary pulley S shall be held by passing a thread several times round it. I gradually produce the current in the primary circuit: I will now suddenly stop this primary current: you observe that the electromotive force is sufficient to break the thread. The inductive effects of one electric circuit upon another depend not alone on the dimensions and form of the two circuits, but on the nature of the material between them. For example, if we had two parallel circular coils, their inductive effects would be very considerably enhanced by introducing a bar of iron in their common axis. We can imitate this effect by moving outwards or inwards these brass weights. In the experiment I have shown you the weights have been some distance from the axis in order to obtain considerable effect, just as in the Ruhmkorff coil an iron core is introduced within the primary circuit. I will now do what is equivalent to removing the core: I will bring the weights nearer to the axis, so that my fly-wheel shall have less moment of inertia. You observe that the inductive effects are very much less marked than they were before. With the same electro-magnet which we used before, but differently arranged, we will show what we have just illustrated—the induction of one circuit on another. Referring to Fig. 4, coil A B corresponds to wheel P; C D to wheel S; and the iron core to the fly-wheel and differential gear. The resistance of a lamp takes the place of the friction of the string on S. As we make and break the circuit you see the effect of the induced current in rendering the lamp incandescent. So far I have been illustrating the phenomena of the induction of one current upon another. I will now show on the model that a current in a single electric circuit has momentum. The secondary wheel shall be firmly held; it shall have no conductivity at all; that is, its electrical effect shall be as

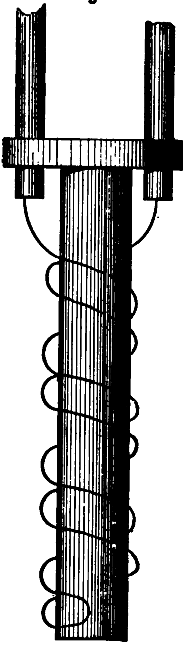
Fig.4



though it were not there. I now cause a current to begin to flow in the primary circuit, and it is obvious enough that a certain amount of work must be done to bring it up to a certain speed. The angular velocity of the fly-wheel is half that of the pulley representing the primary circuit. Now suppose that the two pulleys were connected together in such a way that they must have the same angular velocity in the same direction. This represents the coil having twice as many convolutions as it had before. A little consideration will show that I must do four times as much work to give the primary pulley the same velocity that it attained before; that is to say, that the coefficient of self-induction of a coil of wire is proportional to the square of the number of convolutions. Again, suppose that these two wheels were so geared together that they must always have equal and opposite velocities, you can see that a very small amount of work must be done in order to give the primary wheel the velocity which we gave to it before. Such an arrangement of the model represents an electric circuit, the coefficient of induction of which is exceedingly small, such as the coils that are wound for standard resistances; the wire is there wound double, and the current returns upon itself, as shown in Fig. 5.

In the widest sense, the dynamo-electric machine may be defined as an apparatus for converting mechanical energy into the energy of eloctro-static charge,

Fig.5



or mechanical power into its equivalent electric current through a conductor. Under this definition would be included the electrophorus and all frictional machines; but the term is used, in a more restricted sense, for those machines which produce electric currents by the motion of conductors in a magnetic field, or by the motion of a magnetic field in the neighborhood of a conductor. The laws on which the action of such machines is based have been the subject of a series of discoveries. Oersted discovered that an electric current in a conductor exerted force upon a magnet; Ampère, that two conductors conveying currents generally exerted a mechanical force on each other; Faraday discovered—what Helmholtz and Thomson subsequently proved to be the necessary consequence of the mechanical reactions between conductors conveying currents and magnets—that if a closed conductor move in a magnetic field, there will be a current induced in that conductor in one direction, if the number of lines of magnetic force passing through the conductor was increased by the movement; in the other direction if diminished. Now all dynamo-electric machines are based upon Faraday's discovery. Not only so; but however elaborate we may wish to make the analysis of the action of a dynamo-machine, Faraday's way of presenting the phenomena of electro-magnetism to the mind is in general our best point of departure. The dynamo-machine, then, essentially consists of a conductor made to move in a magnetic field. This conductor, with the external circuit, forms a closed circuit in which electric currents are induced as the number of lines of magnetic force passing through the closed circuit varies. Since, then, if the current in a closed circuit be in one direction when the number of lines of force is increasing, and in the opposite direction when they are diminishing, it is clear that the current in each part of such circuit which passes through the magnetic field must be alternating in direction, unless indeed the circuit be such that it is continually cutting more and more lines of force, always in the same direction. Since the current in the wire of the machine is alternating, so also must be the current outside the machine, unless something in the nature of a commutator be employed to reverse the connections of the internal wires in which the current is induced, and of the external circuit. We have then broadly two classes af dynamo-electric machines; the simplest, the alternating-current machine, where no commutator is used; and the continuous-current machine, in which a commutator is used to change the connection of the external circuit just at the moment when the direction of the current would change. The mathematical theory of the alternate-current machine is comparatively simple. To fix ideas, I will ask you to think of the alternate-current Siemens machine, which Dr. Siemens exhibited here three weeks ago. We have there a series of magnetic fields of alternate polarity, and through these fields we have coils of wire moving; these coils constitute what is called the armature;

in them are induced the currents which give a useful effect outside the machine. Now I am going to trouble you to go through the mathematical equations, simple though they are, by which the following formulæ are obtained:

$$I = A \sin \frac{2\pi t}{T} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(I.)}$$

$$E = \frac{2\pi A}{T} \cos \frac{2\pi t}{T} \quad . \; . \; . \; . \; . \; . \; . \quad \text{(II.)}$$

$$x = \frac{2\pi A}{T} \frac{\cos 2\pi \frac{t - \tau}{T}}{\sqrt{\left(\frac{2\pi\gamma}{T}\right)^2 + R^2}} \quad . \; . \quad \text{(III.)}$$

$$\mathrm{Tan} \frac{2\pi\tau}{T} = \frac{2\pi\gamma}{RT} \quad . \; . \; . \; . \; . \; . \quad \text{(IV.)}$$

$$\psi R \frac{2\pi^2 A^2}{T^2} \frac{I}{\left(\frac{2\pi\gamma}{T}\right)^2 + R^2} \quad . \; . \; . \quad \text{(V.)}$$

$$R = \frac{2\pi\gamma}{T} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(VI.)}$$

T represents the periodic time of the machine; that is, in the case of a Siemens machine having eight magnets on each side of the armature, T represents the time of one-fourth of a revolution. I represents the number of lines of force embraced by the coils of the armature at the time *t*. I must be a periodic function of *t*, in the simplest form represented by Equation I. Equation II. gives E the electromotive force acting at time *t* upon the circuit. Having given the electromotive force acting at any time, it would appear at first sight that we had nothing to do but to divide that electromotive force by the resistance R of the whole circuit, to obtain the current flowing at that time. But if we were to do so we should be landed in error, for the conducting circuit has other properties besides resistance. I pointed out to you that it had a property of momentum represented by its coefficient of self-induction called γ in the formula; and when we are dealing with rapid changes of current, it plays as important a part as the resistance. Formula III. gives the current x, flowing at any time, and you will observe that it shows two things: first the maximum current is less than it would be if there were no self-induction; secondly, it attains its maximum at a later time. This retardation is represented by the letter τ, and its amount is determined by the Formula IV. At a given speed of rotation, the amount of electrical work developed in the machine in any time ψ is given by Formula V. It is greatest when $R=\frac{2\pi\gamma}{T}$. From these formulæ we see that the current is diminished either by increasing γ or increasing R; also the moment of reversal of current is not coincident with the moment of reversal of electromotive force, but occurs later, by an amount depending on the relative magnitudes of γ and R. They show us that although by doubling the velocity of the machine we really double the electromotive force at any time, we do not double the current passing, nor the work done by the machine; but we may see that if we double the velocity of the machine, we may work through double the external resistance, and still obtain the same current. In what precedes, it has been assumed that the copper wires are the only conducting bodies moving in the magnetic field. In many cases the moving wire-coils of these machines have iron cores, the iron being in some cases solid, in others more or less divided. It is found that if such machines are run on open circuit, that is, so that no current circulates in the armatures, the iron becomes hot, very much hotter than when the circuit of the copper wire is closed. In some cases this phenomenon is so marked that the machine actually takes more to drive it, when the machine is on open circuit, than when it is short-circuited. The explanation is that on open circuit currents are induced in the iron cores, but that when the copper coils are closed, the current in them diminishes by induction the current in the iron. The effect of currents in the iron cores is not alone to waste energy and heat the machine; but for a given intensity of field and speed of revolution, the external current produced is diminished. The cure of the evil is to subdivide the moving iron as much as possible, in directions perpendicular to those in which the current tends to circulate.

There remains one point of great practical interest in connection with alternate-current machines. How will they behave when two or more are coupled together, to aid each other in doing the same work? With galvanic batteries, we know very well how to couple them, either in parallel circuit or in series, so that they shall aid, and not oppose, the effects of each other; but with alternate-current machines, independently driven, it is not quite obvious what the result will be, for the polarity of each machine is constantly changing. Will two machines, coupled together, run independently of each other, or will one control the movement of the other in such wise that they settle down to conspire to produce the same effect, or will it be into mutual opposition? It is obvious that a great deal turns upon the answer to this question, for in the general distribution of electric light, it will be desirable to be able to supply the system of conductors from which the consumers draw by separate machines, which can be thrown in and out at pleasure. Now I know it is a common impression that alternate-current machines cannot be worked together, and that it is almost a necessity to have one enormous machine to supply all the consumers drawing from one system of conductors. Let us see how the matter stands. Consider two machines independently driven, so as to have approximately the same periodic time and the same electromotive force. If these two machines are to be worked together, they may be connected in one of two ways; they may be in parallel circuit with regard to the external conductor, as shown by the full line in Fig. 6, that is, their currents may be added algebraically and sent to the external circuit, or they may be coupled in series, as shown by the dotted line, that is, the whole current may pass successively through the two machines, and the electromotive force of the two machines may be added, instead of their currents. The latter case is simpler. Let us consider it first.

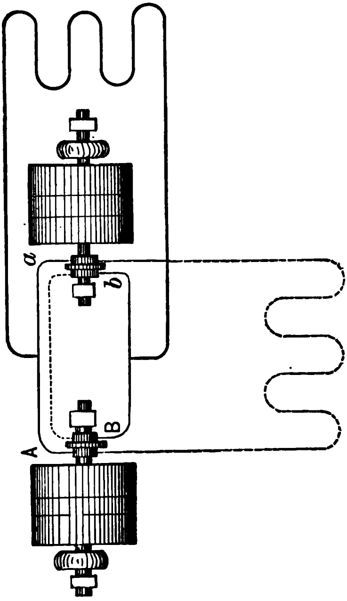


Fig.6

I am going to show that if you couple two such alternate-current machines in series, they will so control each other's phase as to nullify each other, and that you will get no effect from them; and, as a corollary from that, I am going to show that if you couple them in parallel circuit, they will work perfectly well together, and the currents they produce will be added; in fact, that you cannot drive alternate-current machines tandem, but that you may drive them as a pair, or, indeed, any number abreast. In diagram, Fig. 7, the horizontal line of abscissæ represents the time advancing from left to right; the full curves represent the electromotive forces of the two machines not supposed to be in the same phase. We want to see whether they will tend to get into the same phase or to get into opposite phases. Now, if the machines are coupled in series, the resultant electromotive force on the circuit will be the sum of the electromotive forces of the two machines. This resultant electromotive force is represented by the broken curve III; by what we have already seen in Formula IV., the phase of the current must lag behind the phase of the electromotive force, as is shown in the diagram by curve IV., thus———.———.———. Now, the work done in any machine is repre-

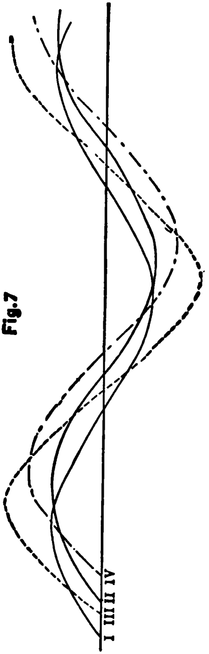


Fig.7

sented by the sum of the products of the currents and of the electromotive forces, and it is clear that as the phase of the current is more near to the phase of the lagging machine II than to that of the leading machine I, the lagging machine must do more work in producing electricity than the leading machine; consequently its velocity will be retarded, and its retardation will go on until the two machines settle down into exactly opposite phases, when no current will pass. The moral, therefore, is, do not attempt to couple two independently driven alternate-current machines in series. Now for the corollary, AB, Fig. 6, represent the two terminals of an alternate-current machine; *ab* the two terminals of another machine independently driven. A and *a* are connected together, and B and *b*. So regarded, the two machines are in series, and we have just proved that they will exactly oppose each other's effects, that is, when A is positive, *a* will be positive also; when A is negative, *a* is also negative. Now, connecting A and *a* through the comparatively high resistance of the external circuit with B and *b*, the current passing through that circuit will not much disturb, if at all, the relations of the two machines. Hence, when A is positive, *a* will be positive, and when A is negative, *a* will be negative also; precisely the condition required that the two machines may work together to send a current into the external circuit. You may, therefore, with confidence, attempt to run alternate-current machines in parallel circuit for the purpose of producing any external effect. I might easily show that the same applies to a larger number: hence, there is no more difficulty in feeding a system of conductors from a number of alternate-current machines, than there is in feeding it from a number of continuous-current machines. A little care only is required that the machine shall be thrown in when it has attained something like its proper velocity. A further corollary is that alternate currents with alternate-current machines as motors may theoretically be used for the transmission of power.*

It is easy to see that, by introducing a commutator revolving with the armature, in an alternate-current machine, and so arranged as to reverse the connection between the armature and the external circuit just at the time when the current would reverse, it is possible to obtain a current always constant in direction; but such a current would be far from constant in intensity, and would certainly not accomplish all the results that are obtained in modern continuous-current machines. This irregularity may, however, be reduced to any extent by multiplying the wires of the armature, giving each its own connection to the outer circuit, and so placing them that the electromotive force attains a maximum successively in the several coils. A practically uniform electric current was first commercially produced with the ring armature of Pacinotti, as perfected by Gramme. The Gramme machine is represented diagrammatically in Fig. 8. The armature consists of an anchor ring of iron wire, the strands more or less insulated from each other. Round this anchor ring is wound a continuous endless coil of copper wire; the armature moves in a magnetic field, produced by permanent or electro-magnets with diametrically opposite poles, marked N and S. The line of magnetic force may be regarded as passing into the ring from N, dividing, passing round the ring and across to S. Thus the coils of wire, both near to N and near to S, are cutting through a very strong magnetic field; consequently there will be an intense inductive action; the inductive action of the coils near N being equal and opposite to the inductive action of the coils near S, it results that there will be strong positive and negative electric potential at the extremities of a diameter perpendicular to the line N S. The

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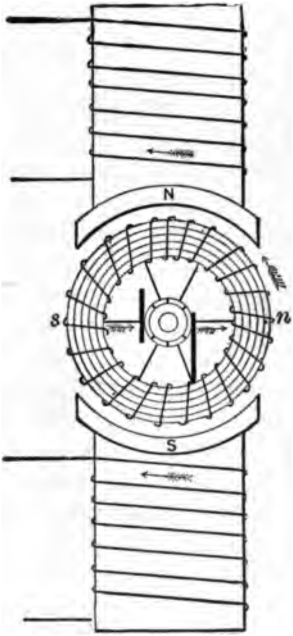
* Of course in applying these conclusions it is necessary to remember that the machines only *tend* to control each other, and that the control of the motive power may be predominant, and *compel* the two or more machines to run at different speeds.

Fig. 8



electromotive force produced, is made use of to produce a current external to the machine thus, the endless coil of the armature is divided into any number of sections, in the diagram into six for convenience, usually into sixty or eighty, and the junction of each pair of sections is connected by a wire to a plate of the commutator fixed upon the shaft which carries the armature; collecting brushes make contact with the commutator as shown in the diagram. If the external resistance were enormously high, so that very little current, or none at all passed through the armature, the greatest difference of potential between the two brushes would be found when they made contact at points at right angels to the line between the magnets; but when a current passes in the armature, this current causes a disturbing effect upon the magnet field. Every time the contact of the brushes changes from one contact-plate to the next, the current in a section of the copper coil is reversed, and this reversal has an inductive effect upon all the other coils of the armature. You may take it from me that the net result on any one coil is approximately the same as if that coil alone were moved, and all the other coils were fixed, and there were no reversals of current in them. Now you can easily see that the magnetic effect of the current circulating in the coils of the armature will be to produce a north pole at *n*, and a south pole at *s*. This will displace the magnetic field in the direction of rotation. If, then, we were to keep the contact points the same as when no current was passing, we should short circuit the sections of the armature at a time when they were cutting through the lines of magnetic force, with a result that there would be vigorous sparks between the collecting brushes and the commutator. To avoid this, the brushes must follow the magnetic field, and also be displaced in the direction of rotation, this displacement being greater as the current in the armature is greater in proportion to the magnetic field. The net effect of this disturbing effect of the current in the armature reacting upon itself is then to displace the neutral points upon the commutator, and consequently somewhat to diminish the effective electromotive force. It is best to adjust the brushes to make contact at a point such that, with the current then passing, flashing is reduced to a minimum, but this point does not necessarily coincide with the point which gives maximum difference of potential. The magnetic field in the Gramme and other continuous dynamo-electric machines, may be produced in several ways. Permanent magnets of steel may be used, as in some of the smaller machines now made, and in all the earlier machines; these are frequently called magneto-machines. Electro-magnets excited by a current from a small dynamo-electric machine, were introduced by Wilde; these may be described shortly as dynamos with separate exciters. The plan of using the whole current from the armature of the machine itself, for exciting the magnets, was proposed almost simultaneously by Siemens, Wheatstone and S. A. Varley. A dynamo, so excited, is now called a series dynamo. Another method is to divide the current from the armature, sending the greater part into the external circuit, and a smaller portion through the electro-magnet, which is then of very much higher resistance; such an arrangement is called a shunt dynamo. A combination of the two last methods has been recently introduced, for the purpose of maintaining constant potential. The magnet is partly excited by a circuit of high resistance, a shunt to the external circuit, and partly by coils conveying the whole current from the armature. All but the first two arrangements named depend on residual magnetism to initiate the current, and below a certain speed of rotation give no practically useful electromotive force. A dynamo machine is, of course, not a perfect instrument for converting mechanical energy into the energy of electric current. Certain losses inevitably occur. There is, of course, the loss due to friction of bearings, and of the collecting brushes upon the commutator; there is also the loss due to the production of electric currents in the iron of the machine.

When these are accounted for, we have the actual electrical effect of the machine in the conducting wire; but all of this is not available for external work. The current has to circulate through the armature, which inevitably has electrical resistance; electrical energy must, therefore, be converted into heat in the armature of the machine. Energy must also be expended in the wire of the electromagnet which produces the field, for the resistance of this also cannot be reduced beyond a certain limit. The loss by the resistance of the wires of the armature and of the magnets greatly depends on the dimensions of the machine. About this I shall have to say a word or two presently. To know the properties of any machine thoroughly it is not enough to know its efficiency and the amount of work it is capable of doing; we need to know what it will do under all circumstances of varying resistance or varying electromotive force. We must know, under any given conditions, what will be the electromotive force of the armature.

Now, this electromotive force depends on the intensity of the magnetic field, and the intensity of the magnetic field depends on the current passing around the electro-magnet and the current in the armature. The current then in the machine is the proper independent variable in terms of which to express the electromotive force. The simplest case is that of the series dynamo, in which the current in the electro-magnet and in the armature is the same, for then we have only one independent variable. The relation between the electromotive force and current is represented by such a curve as is shown in the diagram, Fig. 9. The abscissæ, measured along OX, represent the current, and the ordinates represent the electromotive force in the armature. When four years ago I first used this curve, for the purpose of expressing the results of my experiments on the Siemens dynamo machine, I pointed out that it was capable of solving almost any problem relating to a particular machine, and that it was also capable

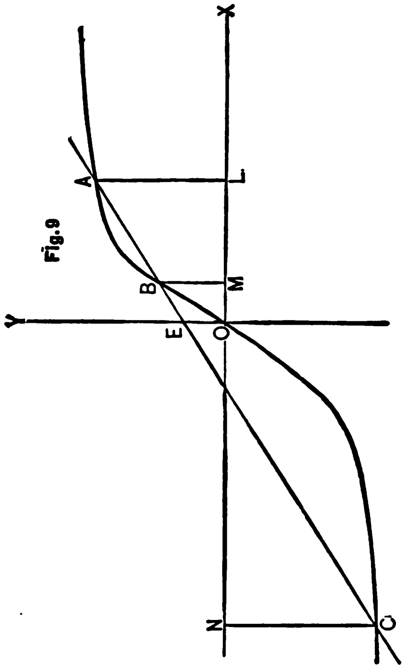


Fig. 9

of giving good indications of the results of changes in the winding of the magnets, or of the armatures of such machines. Since then M. Marcel Deprez has happily named such curves "characteristic curves." I will give you one or two illustrations of their use. A complete characteristic of a series dynamo does not terminate at the origin, but has a negative branch, as shown in the diagram; for it is clear that by reversing the current through the whole machine, the electromotive force is also reversed. Suppose a series dynamo is used for charging an accumulator, and is driven at a given speed, what current will pass through it? The problem is easily solved. Along OY, Fig. 9, set off OE to represent the electromotive force of the accumuluator, and through E draw the line CEBA, making an angle with OX, such that its tangent is equal to the resistance of the whole circuit, and cutting the characteristic curve as it in general will do, in three points A, B and C. We have then three answers to the question. The current passing through the dynamo will be either OL, OM, or ON, the abscissæ of the points where the line cuts the curve. OL represents the current when the dynamo is actually charging the accumulator. OM represents a current which could exist for an instant, but which would be unstable, for the least variation would tend to increase. ON is the current which passes, if the current in the dynamo should get reversed, as it is very apt to do when used for this purpose. The next illustration is rather outside my subject, but shows another method of using the characteristic curve. Many of you have heard of Jacobi's law of maximum effect of transmitting work by dynamo machines. It is this: Supposing that the two dynamo machines were perfect instruments for converting mechanical energy into electrical energy, and that the generating machine were run at constant velocity, whilst the receiving machine had a variable velocity, the greatest amount of work would be developed in the receiving machine when its electromotive force was one-half that of the generating machine, then the efficiency would be one-half, and the electrical work done by the generating machine would be just one-half of what it would be if the receiving machine were forcibly held at rest. Now this law is strictly true if, and only if, the electromotive force of the generating machine is independent of the current passing through its armature. What I am now going to do is to give you a construction for determining the maximum work which can be transmitted when the electromotive force of the generating machine depends on the current passing through the armature, as, indeed, it nearly always does. Referring to Fig. 10. OPB is the characteristic curve of the generating machine; construct a derived curve thus, at successive points P of the characteristic curve, draw tangents PT, draw TN parallel to OX, cutting PM in N, produce MP to L, making LP equal PN; the point L gives the derived curve, which I want. Now, to find the maximum work which can be transmitted, draw OA at such angle with OX that its tangent is equal to twice the resistance of the whole circuit, cutting the derived

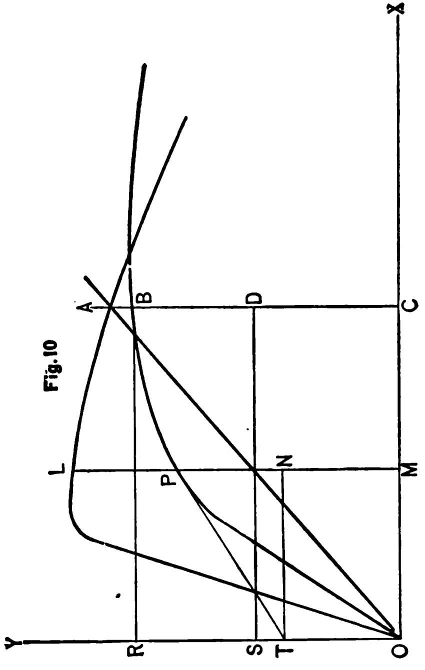


Fig. 10

curve in A. Draw the ordinate AC, cutting the characteristic curve in B; bisect AC at D. The work expended upon the generating machine would be represented by the parallelogram OCBR, the work wasted in resistance by OCDS, and the work developed in the receiving machine by the parallelogram SDBR.

When the dynamo-machine is not a series dynamo, but the currents in the armature and in the electro-magnet, though possibly dependent upon each other are not necessarily equal, the problem is not quite so simple. We have, then, two variables, the current in the electro-magnet and the current in the armature; and the proper representation of the properties of the machine will be by a characteristic surface such as that illustrated by this model, Fig. 11. Of the three co-ordinate axes, OX represents the current in the magnet; OY represents the current in the armature not necessarily to the same scale, and OZ the electromotive force. By the aid of such a surface as this one may deal with any problem relating to a dynamo-machine, no matter how its electro-magnets and its armature are connected together. Let us apply the model to find the character-

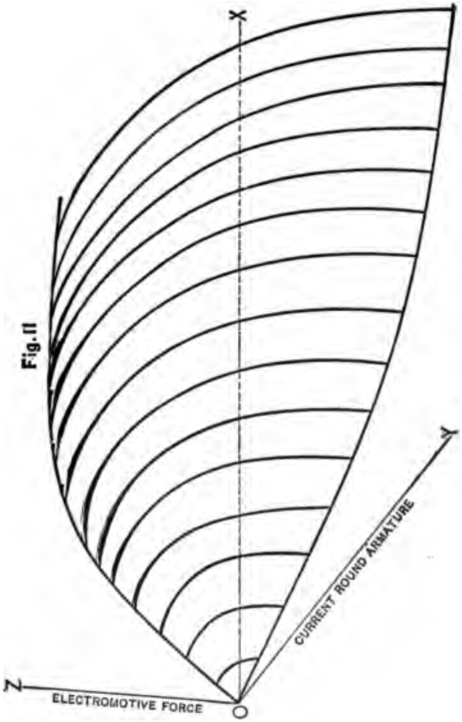


Fig. II

istic of a series dynamo. Take a plane through OZ, the axis of electomotive force, and making such an angle with the plane OX, OZ, that its tangent is equal to current unity on axis OY, divided by current unity on axis OX. This plane cuts the surface in a curve. The projection of this curve on the plane OX, OZ is the characteristic curve of the series dynamo. This model only shows an eighth part of the complete surface. If any of you should interest yourselves about the other seven parts, which are not without interest, remember that it is assumed that the brushes always make contact with the commutator at the point of no flashing, it there is one. Of course in actual practice one would not use the model of the surface, but the projections of its sections. While I am speaking of characteristic curves there is one point I will just take this opportunity of mentioning. Three years ago, Mr. Shoolbred exhibited the characteristic curve of Gramme machine, in which, after the current attained to a certain amount, the electromotive force began to fall. I then said that I thought there must be some mistake in the experiment. However, subsequent experiments have verified the fact; and when one considers it, it is not very difficult to see the explanation. It lies in this: after the current attains to a certain amount the iron in the machines becomes magnetically nearly saturated, and consequently an increase in the current does not produce a corresponding increase in the magnetic field. The reaction, however, between the different sections of the wire on the armature goes on increasing indefinitely, and its effect is to diminish the electromotive force.

A little while ago I said that the dimensions of the machine had a good deal to do with its efficiency. Let us see how the properties of a machine depend upon its dimensions. Suppose two machines alike in every particular excepting that the one has all its linear dimensions double that of the other; obviously enough all the surfaces in the larger would be four times the corresponding surfaces in the smaller, and the weights and volumes of the larger would be eight times the corresponding weights in the smaller machines. The electrical resistances in the larger machine would be one-half of the smaller. The current required to produce a given intensity of magnetic field would be twice as great in the larger machine as in the smaller. In the diagram, Fig. 12, are shown the comparative characteristic curves of the two machines, when driven at the same speed. You will observe that the two curves are one the projection of the other, having corresponding points with abscissæ in the ratio of one to two, and the ordinates in the ratio of one to four. Now at first sight it would seem as though, since the wire on the magnet and armature of the larger machine has four times the section of that of the smaller, that four times the current could be carried, that consequently the intensity of the magnetic field would be twice as great, and its area would be four times as great, and hence the electromotive force eight times as great; and since the current in the arma-

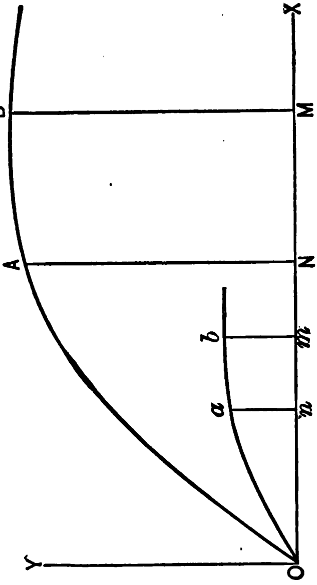


Fig. 12

ture also is supposed to be four times as great, that the work done by the larger machine would be thirty-two times as much as that which would be done by the smaller. Practically, however, no such result can possibly be attained, for a whole series of reasons. First of all, the iron of the magnets becomes saturated, and consequently instead of getting eight times the electromotive force, we should only get four times the electromotive force. Secondly, the current which we can carry in the armature is limited by the rate at which we can get rid of the heat generated in the armature. This we may consider as proportional to its surface, consequently we must only waste four times as much energy in the armature of the larger machine as in the smaller one, instead of eight times, as would be the case if we carried the current in proportion to the section of the wire. Again, the larger machine cannot run at so great an angular velocity as the smaller one. And lastly, since in the larger machine the current in the armature is greater in proportion to the saturated magnetic field than it is in the small one, the displacement of the point of contact of the brushes with the commutator will be greater. However, to cut the matter, about which one might say a great deal, short, one may say that the capacity of similar dynamo-machines is pretty much proportionate to their weight, that is, to the cube of their linear dimensions; that the work wasted in producing the magnetic field will be directly as the linear dimensions; and that the work wasted in heating the wires of the armature will be as the square of the linear dimensions. Now let us see how this would practically apply. Suppose we had a small machine capable of producing an electric current of 4 HP., that of this 4 HP. 1 was wasted in heating the wires of the armature, and 1 in heating the wires of the magnet, 2 would be usefully applied outside. Now, if we doubled the linear dimensions we should have a capacity of 32 HP., of which 2 only, if suitably applied, would be required to produce the magnetic field and 4 would be wasted in heating the wires of the armature, leaving 26 HP. available for useful work outside the machine—a very different economy from that of the smaller machines. But if we again doubled the linear dimensions of our machine, we should by no means obtain a similar increase of effect. A consideration of the properties of similar machines has another very important practical use. As you all know, Mr. Froude was able to control the design of ironclad ships by experiments upon models made in paraffin wax. Now, it is a very much easier thing to predict what the performance of a large dynamo-machine will be, from laboratory experiments made upon a model of a very small fraction of its dimensions. As a proof of the practical utility of such methods, I may say that by laboratory experiments I have succeeded in increasing the capacity of the Edison machines without increasing their cost, and with a small increase of their percentage of efficiency, remarkably high as that efficiency already was.

I might occupy your time with considerations as to the proper proportion of conductors, and explain Sir W. Thomson's law, that the most economical size of a copper conductor is such that the annual charge for interest and depreciation of the copper of which it is made, shall be equal to the cost of producing the power which is wasted by its resistance. But the remaining time will, perhaps, be best spent in considering the production of light from the energy of electric currents. You all know that this is done commercially in two ways, by the electric arc, and by the incandescent lamp; as the arc lamp preceded the incandescent lamp historically, we will examine one or two points connected with it first.

I have here all that is necessary to illustrate the electric arc, viz., two rods of carbon supported in line with each other, and so mounted that they can be approached or withdrawn. Each carbon is connected with one of the poles of the Edison dynamo-machine, which is supplying electricity to the incandescent lamps which illuminate the whole of this building. A resistance is interposed in the circuit of the lamp, because the electromotive force of the machine is much in excess of what the lamp requires. I now approach the carbons, bring them into contact, and again separate them slightly; you observe that the break does not stop the current which forces its way across the space. I increase the distance between the carbons, and you observe the electric arc between their extremities; at last it breaks, having attained a length of about one inch. Now the current has hard work to cross this air-space between the carbons, and the energy there developed is converted into heat, which raises the temperature of the ends of the carbon beyond any other terrestrial temperature. There are several points of interest I wish to notice in the electric arc. Both carbons burn away in the air, but there is also a transference of carbon from the positive to the negative carbon; therefore, although both waste away, the positive carbon wastes about twice as fast as the negative. With a continuous current, such as we are using now, the negative carbon becomes pointed, whilst the positive carbon forms a crater or hollow; it is this crater which becomes most intensely hot and radiates most of the light, hence the light is not by any means uniformly distributed in all directions, but is mainly thrown forward from the crater in the positive carbon. This peculiarity is of great advantage for some purposes, such, for example, as military or naval search lights, but it necessitates, in describing the illuminating power of an arc light, some statement of the direction in which the measurement was made. On account of its very high temperature, the arc light sends forth a very large amount of visible radiation, and is therefore very economical of electrical energy. For the same reason its light contains a very large proportion of rays of high refrangibility, blue and ultra-violet. I have measured the red light of an electric arc against the red of a candle, and have found it to be 4,700 times as great, and I have measured the blue of the same arc light against the blue of the same candle, and found it to be 11,380 times as great. The properties of an electric arc are not those of an ordinary conductor. Ohm's law does not apply. The electromotive force and the current do not by any means bear to each other a constant ratio. Strictly speaking, an electric arc cannot be said to have an electric resistance measureable in ohms. We will now examine the electrical properties of the arc experimentally. In the circuit with the lamp is a Thomson graded current galvanometer for measuring the current passing in amperes; connected to the two carbons is a Thomson graded potential galvanometer, for measuring the difference of potential between them in volts. We have the means of varying the current by varying the resistance, which I have already told you is introduced into the circuit. We will first put in circuit the whole resistance available, and will adjust the carbons so that the distance between them is, so near as I can judge, $\frac{1}{8}$ inch. We will afterwards increase the current, and repeat the readings. The results are given in the following table:

Current galvanometer.	Potential galvanometer.	Ampères.	Volts.	Watts.	HP.
6.2	12.0	9.9	35	346	0.46
9.3	12.0	14 9	35	521	0.70
11.5	11.8	18.4	34	626	0.84

If the electrical properties of the arc were the same as those of a continuous conductor, the volts would be in proportion to the amperes, if correction were made for change of temperature; you observe that instead of that the potential is nearly the same in the two cases. We may say, with some approach to accuracy, that with a given length of arc the arc opposes to the current an electromotive force nearly constant, almost independent of the current. This was first pointed out by Edlund. If you will speak of the resistance of the electric arc, you may say that the resistance varies inversely as the current. Take the last experiment: by burning 4 cubic feet of gas per hour we should produce heat-energy at about the same rate. I leave any of you to judge of the comparative illuminating effects. It is not my purpose to describe the mechanisms which have been invented for controlling the feeding of the carbons as they waste away. Several lamps lent by Messrs. Siemens Brothers—to whom I am indebted for the lamp and resistance I have just been using—lie upon the table for inspection. An electric arc can also be produced by an alternate current. Its theory may be treated mathematically, and is very interesting, but time will not allow us to go into it. I will merely point out this: there is some theoretical reason to suppose that an alternate-current arc is in some measure less efficient than one produced by a continuous current. The efficiency of a source of light is greater, as the mean temperature of the radiating surface is greater. The maximum temperature in an arc is limited probably by the temperature of volatilization of carbon; in an alternate-current arc the current is not constant, therefore the mean temperature is less than the maximum temperature;

in a continuous-current arc, the current being constant, the mean and maximum temperatures are equal, therefore in a continuous-current arc the mean temperature is likely to be somewhat higher than in an alternate-current arc.

We will now pass to the simpler incandescent light. When a current of electricity pass through a continuous conductor, it encounters resistance, and heat is generated, as was shown by Joule, at a rate represented by the resistance multiplied by the square of the current. If the current is sufficiently great the heat will be generated at such a rate that the conductor rises in temperature so far that it becomes incandescent and radiates light. Attempts have been made to use platinum and platinum-iridium as the incandescent conductor, but these bodies are too expensive for general use, and besides, refractory though they are, they are not refractory enough to stand the high temperature required for economical incandescent lighting. Commercial success was not realized until very thin and very uniform threads or filaments of carbon were produced and enclosed in reservoirs of glass, from which the air was exhausted to the utmost possible limit. Such are the lamps made by Mr. Edison with which this building is lighted to-night. Let us examine the electrical properties of such a lamp. Here is a lamp intended to carry the same current as those overhead, but of half the resistance, selected because it leaves us a margin of electromotive force wherewith to vary our experiment. Into its circuit I am able to introduce a resistance for checking the current, composed of other incandescent lamps for convenience, but which I shall cover over that they may not distract your attention. As before, we have two galvanometers, one to measure the current passing through the lamp, the other the difference of potential at its terminals. First of all we will introduce a considerable resistance; you observe that, although the lamp gives some light, it is feeble and red, indicating a low temperature. We take our galvanometer readings. We now diminish the resistance, the lamp is now a little short of its standard intensity; with this current it would last 1000 hours without giving way. We again read the galvanometers. The resistance is diminished still further. You observe a great increase of brightness, and the light is much whiter than before. With this current the lamp would not last very long. The results are given in the following table:

Current galvanometer.	Potential galvanometer.	Ampères.	Volts.	Watts	Resistance ohms.
5.2	12.8	0.38	37	14	97
6 0	14.3	0.44	41	18	93
11.5	23.4	0.84	68	57	81

There are three things I want you to notice in these experiments: first, the light is whiter as the current increases; second, the quantity of light increases very much faster than the power expended increases; and thirdly, the resistance of the carbon filament diminishes as its temperature increases, which is just the opposite of what we should find with a metallic conductor. This resistance is given in ohms in the last column. To the second point, which has been very clearly put by Dr. Siemens in his British Association address, I shall return in a minute or two.

The building is this evening lighted by about 200 lamps, each giving sixteen candles' light, when 75 watts of power are developed in the lamp. To produce the same sixteen candles' light in ordinary flat-flame gas-burners, would require between seven and eight cubic feet of gas per hour, contributing heat to the atmosphere at the rate of 3,400,000 foot-pounds per hour, equivalent to 1250 watts, that is to say, equivalent gas lighting would heat the air nearly seventeen times as much as the incandescent lamps.

Look at it another way. Practically, about eight of these lamps take one indicated horse-power in the engine to supply them. If the steam engine were replaced by a large gas engine this 1 HP. would be supplied by 25 cubic feet of gas per hour, or by rather less therefore by burning gas in a gas engine driving a dynamo, and using the electricity in the ordinary way in incandescent lamps, we can obtain more than 5 candles per cubic foot of gas, a result you would be puzzled to obtain in 16-candle gas burners. With arc lights instead of incandescent lamps many times as much light could be obtained.

At the present time, lighting by electricity in London must cost something more than lighting by gas. Let us see what are the prospects of reduction of this cost. Beginning with the engine and boiler, the electrician has no right to look forward to any marked and exceptional advance in their economy. Next comes the dynamo, the best of these are so good, converting 80 per cent. of the work done in driving the machine into electrical work outside the machine, that there is little room for economy in the conversion of mechanical into electrical energy; but the prime cost of the dynamo-machine is sure to be greatly reduced. Our hope of greatly increased economy must be mainly based upon probable improvements in the incadescent lamp, and to this the greatest attention ought to be directed. You have seen that a great economy of power can be obtained by working the lamps at high-pressure, but then they soon break down. In ordinary practice, from 140 to 200 candles are obtained from a horse-power developed in the lamps, but for a short time I have seen over 1000 candles per horse-power from incandescent lamps. The problem, then, is so to improve the lamp in details, that it will last a reasonable time when pressed to that degree of efficiency. There is no theoretical bar to such improvements, and it must be remembered that incandescent lamps have only been articles of commerce for about three years, and already much has been done. If such an improvement were realized, it would mean that you would get five times as much light for a sovereign, as you can now. As things now stand, so soon as those who supply electricity have reasonable facilities for reaching their customers, electric lighting will succeed commercially where other considerations than cost have weight. We are sure of some considerable improvement in the lamps, and there is a probability that these improvements may go so far as to reduce the cost to one-fifth of what it now is. I leave you to judge whether or not it is probable, nay, almost certain, that lighting by electricity is the lighting of the future.

II.

ON THE MEASUREMENT OF ELECTRICITY

—FOR—

COMMERCIAL PURPOSES.

BY

JAMES N. SHOOLBRED,

B. A., Mem. Inst. C. E.

ON THE MEASUREMENT OF ELECTRICITY FOR COMMERCIAL PURPOSES.*

THE present remarks are intended to deal more especially with the commercial question, raised by what is termed the "supply of electricity," in the Electric Lighting Act, 1882, the objects of such supply being more particularly lighting for public and private purposes, and the transmission of power. All these require electric currents of such magnitude as are generally produced by dynamo machines. To such branches of the subject as telegraphy and telephony, excepting in the general principles common to electric currents, the scope of the paper, and especially the special form of the various apparatus described, will but indirectly apply. The remarks must, in fact, be taken as mainly referring to an electric supply by mechanically-generated induction currents. The basis and principle upon which rests this mode of producing electricity is the conversion of mechanical into electrical energy. Any exact financial evaluation of the commodity produced and offered for sale, involves a knowledge, by measurement, of the amount of this energy, not merely of the electrical energy as offered to the customer, but also of the mechanical which is expended and absorbed in such offer. The former is a question mainly for the consumer, the latter for the producer, who must, however, check the two together to enable him to arrive at the fair marketable value of the supply which is offered for sale. In most cases the evaluation of the mechanical energy expended resolves itself into coal, wages, engine-room expenses, outside working and collection charges, interest on capital expended, and depreciation on plant employed, but chiefly on the two first-named items—coal and wages. It is not intended in these remarks to deal with this portion of the subject, which is one of statistics, but with the more pertinent one, to this society at least, of how to measure and value the electrical product resulting from the above expenditure. Before dismissing the former, the mechanical branch of the subject, it may be well to advert to the similarity, in most cases, and taken broadly, in the nature of the conditions of manufacture and of supply of electricity and of gas, both for illumination and for power. Each starts with coal, the energy of which it transforms into a commodity for the purposes just named; and it is in the process of transformation, in the nature and costliness of the intermediate operations, and in the amount and value of the plant and working expenses involved in each system respectively, that the main commercial difference between the two must exist. Iron forms a large part of the material of the plant necessary in both, and coal and wages, subject to the same conditions and variations, form the main bulk of the working expenses of the two systems. This similarity in objects and in operations is here referred to, in order to point out that, as it is by the careful experience of more than half a century, the conditions, both legal and social, have been ascertained for the supply of gas—and also, it should be added of water—it need not, therefore, be surprising if some of these conditions, stamped with the seal of every-day practice, should be found to adapt themselves, or to be worth while taking into consideration, in the case of electricity, the differences in the processes of production, and of the products themselves, being always remembered. The valuation of the electrical energy of the supply depends upon the exact measurement of two factors—the amount of the supply and the pressure under which it is given, or, in other words, the quantity of the current, and the difference of potential—or electromotive—force which the consumer is able to avail himself of. As there appears to exist a misconception, in some quarters, as to the exact terms in which the various electrical units have been defined, and the precise duty of each, more especially as to the "ampère" and the "coulomb"—for each of which the term "weber" was formerly and almost indiscriminately used—it may be well to give the sense of the resolutions passed at the International Congress of Electricians, held in Paris in 1881, and now universally accepted.

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*Read before the Society of Telegraph Engineers and Electricians.

Electrical units. — The fundamental units adopted are those of the centimeter —length—the gramme—mass—and the second—time—or, as it is called for brevity, the C.G.S. system. For practical units the following are adopted: The ohm—10^9 C.G.S. units—as the unit of resistance, is that of a column of mercury having one square millimeter of section, and of a length hereafter to be determined by a commission specially appointed for the purpose. *Note.*—The length is supposed, however, to be between 104 and 105 centimeters. The volt—10^8 C.G.S. units of electro-motive force—as the unit of electro-motive force. *Note.*—This corresponds nearly to that the product of volts × amperes × time; or, by the product of volts × coulombs, or, expressed algebraically, $W = E\,C\,t = E\,Q$. The following are the equivalent expressions for the same amount of energy, only expressed in other terms, some of which may, perhaps, be more familiar:

Energy expended per second.

$$\text{Volt} \times \text{ampere} = \begin{cases} 1 \text{ watt.} \\ \frac{1}{1\cdot 35} \text{ foot-pounds.} \\ \frac{1}{9\cdot 81} \text{ kilogrammeters.} \\ \frac{1}{736} \text{ force cheval — French horse-power.} \\ \frac{1}{746} \text{ horse-power indicated.} \end{cases}$$

It is suggested in some of the—draft—provisional orders for the supply of electricity, now before the Board of Trade, that the unit of price to be charged should be based "on the energy contained in a current of 1,000 amperes, flowing under an electro-motive force of one volt during one hour;" or, in other words, the unit might be put as 1,000 volt-ampere-hours. Since the ampere hour is another way of saying 3,600 coulombs of quantity of electricity supplied, this above expression may be put thus, as representing the value of the above unit.

Work done.

1000 volt-ampere-hours =
3,600,000 volt-coulombs.
1000 watt's hours.
1.34 horse-power hours.
2,653,200 foot-pounds.
1.36 force cheval heures—French horse-power hours nearly.
100 kilogrammeters.

Put in terms more in accordance with actual practice, the above unit might mean the supply for one hour of a current of ten amperes with an electro-motive force of 100 volts. To arrive at a due evaluation of the supply of electric energy, it is evident that the measurement of each of these two factors—in volts and amperes respectively—should be effected either separately or combined; and also that a continuous and cumulative record should be kept of the supply as it proceeds. Many instruments exist for the measurement of the above elements at any particular time, but without any means of continuously recording such measurements. These evidently, except under certain conditions, cannot comply with the commercial conditions required for ascertaining the amount of supply. It is only, therefore, instruments furnished with means of continuously integrating, or recording the successive progressions of the supply, or instruments to which such recording apparatus can readily be attached, that come properly within the scope of this paper. To measure with completeness for commercial purposes a supply of electricity will entail, therefore, a continuous record of each of the two elements just referred to, current and pressure, either separately or combined. In the supply of towns, however, the question for the consumer, may, and will most probably, be much simplified by causing one of these elements, that of pressure, to remain constant, since it is very likely that a constant standard pressure of supply will be fixed by the Government in granting the several provisional orders. If so, it then becomes the duty of the suppliers to keep up to that pressure under penalty; and instruments for recording such pressure will have to be installed where required, and placed under proper supervision. For the customer, however, it will then generally suffice to have an exact record of the quantity only of his individual consumption of electric supply. It has been thought advisable to precede the description of the recording or registering instruments, which alone are meters in the commonly-accepted sense of the word, by an enumeration of some of those non-recording instruments which are in more general use, since they suffice for present exigencies. This short descriptive enumeration is even almost necessary, since most of these instruments, by the addition of some recording apparatus or appliance by which the element of time can be integrated, may be made to enter into the class of registering meters. Indeed, some of them already possess their representative in this second class, or else have given rise to some modification, which has complied with the requirements in the latter case. Thus, any current or ampère measurer may be converted into a record of quantity, or a coulomb meter, by the integration of the time during which the current has flowed; and similarly, any power or volt ampere measurer may become a register of work done by means of the addition of the elements of time. Again, volt or pressure measurers will always be required in any case where a check is required upon the actual difference of potentials, or electro-motive force of the supply, and this may arise from a variety of causes.

Non-registering instruments. — Current meters: Siemens' electro dynamometer;* Professors Ayrton and Perry, ammeter;* Sir Wm. Thomson, current galvanometer; Capt. Cardew, R.E., low resistance galvanometer;* Dr. Obach, tangent galvanometer;* Marcel Deprez, galvanometer. Pressure meters: Professors Ayrton and Perry, volt-meter;

Sir Wm. Thomson, potential galvanometer; Siemens' torsion galvanometer; Marcel Deprez, galvanometer. Power, or energy, meters: Professors Ayrton and Perry, power meter; C. V. Boys, power meter.

Registering meters. — Quantity, or coulomb, meters: (1) Electrolytic: T. A. Edison, total deposition; T. A. Edison, alternations of deposition; J T. Sprague, alternations of deposition. (2) Mechanical: Dr. J. Hopkinson, rotating meter; C. V. Boys, vibrating meter; Theos. Varley and Greenwood, disc meter; Professors Ayrton and Perry, fluid friction on magneto-electro motor;* T. A. Edison, fluid friction on magneto-electric motor.† Energy, or work, meters: Professors Ayrton and Perry erg-meter; C. V. Boys, energy meter; Deprez-Abdank, energy meter.

Edison's current meters.—These are based upon electro-deposition of metal, due to the action of a known fractional part of the total current. The weight of the increments is ascertained periodically, and from it the total quantity of the current which has passed during the interval is deduced. The metal now generally used consists of plates of zinc, immersed in a solution of ninety parts of sulphate of zinc and one hundred parts of pure water. In the form of meter for commercial use, two cells are placed as a check against one another; one, termed the "monthly cell," receiving four times the current of the other, which is known as the "quarterly cell." In order to prevent the temperature of the liquid in the cells falling so low as to freeze, a connection is made by means of a long thin strip of brass and steel riveted together to an incandescent lamp, which is thereby lighted and raises the temperature as required. It is only when the temperature falls to 42° Fah. that this tongue is sufficiently depressed to form contact, and so to light the lamp. On the temperature rising the tongue rises, and the lamp is extinguished. Experience shows that electro-deposition, to give a true and reliable record, should not be forced or overworked in its action; and that the plates should not in their daily duty be required to do more work, or be longer in action than they are intended for by their superficial area. In practice, about 75 per cent. only of their nominal work should be required of them. Several other forms of meter have been devised by Edison, such as his beam meter, where, when the increments by electro-deposition have accumulated to a certain limit, the current is reversed and the accumulation is re-dissolved, to re-commence again when the normal condition is gained; and, again, electric-motor meters, by fluid pressure, etc. All of these have, it is said, in practice been found—in cases where the total current supplied is very small, and this often so—to require so large a proportion of it for these mechanical operations, as to make the record unreliable. The meter first described, by electrolytic action only, is therefore now generally adopted by Edison.

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* See "Journal" Soc. Tel. Engrs.—No. 43, vol. xi Sept. 1882.

† See "Journal" Soc. Tel. Engrs., Sept. 1882.

Sprague's meters.— These instruments are based upon electro-deposit up to a certain point; *i. e.*, when the intended quantity of metal, whether copper or zinc, has been deposited on the plate. The current is then reversed, and the metal gradually dissolved again until the primary condition of the plate is reached; when, by another reversal of the current, deposition again commences; each reversal of the current being recorded by a mechanical counter and a train and wheels. Not much practical experience has so far been obtained with these meters; but, what has been done tends to point out that the mechanical operations involved in the reversals of the current, and in their registration, absorb a large amount of power.

Hopkinson's current meter.— This instrument consists of a thick wire coil, in the form of a solenoid, through which the current passes to be measured. The iron core of this solenoid revolves with its central shaft by the action of the armature of a small dynamo machine, placed at one end of the shaft. The core of the solenoid is in two parts; the lower is fixed to the shaft, while the upper is movable, being attached to a governor ball arrangement, and sliding up and down the shaft in accordance with the variations in the rotation speed of the shaft. A shunt current passes through the dynamo and its armature, then up through the lower or fixed portion of the core, and—by contact only—to the sliding part, and thence to the framework of the apparatus. If the movable core be lifted, owing to the speed of rotation by the action of the governor balls, this circuit is broken, and the shunt current through the dynamo interrupted. Whenever a current to be measured passes through the coil, attraction, by means of its casing, takes place between the fixed and the movable parts of the iron core. This magnetic action, which is proportional to the square of the current, tends to keep the two parts of the core together and in contact, while the centrifugal force of the governor balls, which is proportional to the square of the speed of revolution, tends to break the contact by lifting the movable part. These opposite forces will, in working, balance one another, and the result is that the system revolves with a velocity proportional to the current through the coil. As the revolutions of the shaft are transmitted continuously by a train of wheels to a set of index dials, a record is thus kept of the quantity of the current that has passed. In the construction of the apparatus the weight of the core is taken off by springs. The arrangement of the parts, as now made, is shown on the diagram.

Boys' quantity or vibrating meter.—This instrument is based upon two well-known principles. (1) The force acting on the armature of an electro-magnet, in any position, is proportional to the square of the current. (2) The square of the number of vibrations, say, of a pendulum, is a measure of the controlling force.

Therefore, if the controlling force under which a body vibrates is due to the action of an electro-magnet on its armature, the square of the number of vibrations in a given time is a measure of the square of the electric current. In other words, the rate of vibrating is a measure of the strength of the current, and the number of vibrations is a measure of its quantity. The exact form and nature of the meter may vary in many details. The one now shown consists, primarily, of an electro-magnet—the upper one in the diagram—through the coils of which passes a portion of the main current to be measured. This magnet is placed horizontally, and a vertical rocking shaft stands between its poles. This shaft has fixed on it a soft iron armature, rounded at the ends, and free to move in the horizontal plane between the poles of the electro-magnet. The intensity of the attraction between the poles and this armature determines the rate of vibration; which, as above-stated, is a measure of the strength of the current. Each vibration is itself recorded by means of an escapement, a train of wheels, and a set of index dials; and the number of vibrations thus registered becomes a measure of the quantity of the current. To add to the momentum of the vibrating body, two long arms, weighted at the end, are attached to the lower part of the vertical shaft. In order, likewise, to prevent the vibrating armature from gradually coming to rest, it is arranged that, when the vibrations fall below a certain limit, by making contact, a portion of the current is sent round the coils of a second or "impulse" electro-magnet—placed underneath the "controlling" magnet—and which has an armature of a suitable form fixed on to the same shaft that carries the armature of the upper magnet. The extra motion thus given to the shaft, by the attraction of the lower armature, affords the necessary impulse to the vibrating armature when required to do so.

Boys' energy meter.—This instrument consists of two parts: (1) the indicator of energy, and (2) the integrating apparatus. (1) In the indicator of energy, a balanced beam has from one end suspended a counterweight, and from the other a hollow solenoid, free to work up and down into two other fixed solenoids. The movable solenoid is wound with a considerable length of fine wire; in the upper half in one direction, in the lower in the opposite—this is to render it independent of any magnet which may be placed near to it. This solenoid constitutes the high resistance shunt which measures the electro-motive force. The two fixed solenoids are wound with thick wire, and convey the main current. The result of the action of the fixed and the movable solenoids on each other is a force proportional to the product of the two currents, that is the energy being expended, but the external evidence of this is the inclination of the beam, and this inclination, or rather the tangent of the inclination, is proportional to the energy being expended. (2) The recording apparatus consists of a cylinder, which by means of a mangle motion is made to reciprocate backwards and forwards by clockwork, and during its passage in each direction the cylinder is made to bear alternately against one of two tangent wheels, each free to be inclined in its direction of travel; both are fixed on the same swivelling frame, but only one of them bears at the same time against the cylinder. This frame is free to be inclined from the vertical in correspondence with the inclinations of the beam above mentioned; but the tangent of the inclination of the beam, as has been said, is proportional to the energy of the current; so also, therefore, is the tangent of the inclination of the wheels $\left(i.\ e., \frac{dy}{dx}\right)$ The effect of this inclination of the tangent wheel is to cause the reciprocating cylinder to rotate, the speed of such rotation being proportional to the tangent of the inclination of the wheel, which is likewise proportional to the tangent of the inclination of the beam; that is, to the amount of energy expended. The path of the tangent wheel on the reciprocating cylinder, when not inclined, is simply a straight line lengthways along the cylinder, and no rotation is caused, but when, owing to the · inclination of the wheel, the cylinder rotates, the wheel pathos becomes a spiral. The rotations of the cylinder are transmitted to a train of wheels and registered, thus giving a record of the amount of energy expended during a given time.

Ayrton and Perry's erg-meter.—This instrument is but a further development or sequel to their power meter, by the addition of apparatus which integrate and record continuously the time during which the electrical energy has been imparted, as well as the variations in its amount. By this means is preserved a record of the entire work done, or of the total electrical energy supplied. As in the power meter, two coils are here made use of, a thick wire one on the main circuit, to measure the amount of current, and a thin wire one on a shunt joining the ends of the main circuit, to measure the difference of potentials or electro-motive force of the main circuit. In the arrangement, as now shown, the thin wire coil, of say one 1,000 ohms resistance, simply replaces the pendulum bob of a clock. The wires from each end of the coils pass up the sides of the pendulum rod and on to the binding screws A and B, which can be joined to the supply and return cables of a house, or machine, or a system receiving electrical energy. In the immediate vicinity of the fine wire coil, fixed to the clock case, and parallel with the plane of the pendulum path, is fixed the thick wire coil, which forms part of the main circuit, and has a very small resistance. The effect upon the thin wire coil of its repeated passages in front of the thick wire coil is to cause a certain pull or attraction upon its motion —either of acceleration or of retardation, according to the direction of the coiling. This acting, in addition to the ordinary action of gravity upon the pendulum, will keep constantly adding to, or retarding, its rate of motion in proportion to the electrical power of the circuit. This pull is the product of the magnetic moments of the two coils, and therefore is proportional to the product of the current and the electro-motive force. The effects of these repeated accelerations or retardations upon the progress of the clock keep constantly accumulating, and their total amount can at any time be detected and ascertained by observing the amount of loss or gain which the clock has experienced. As the rate of loss or gain in the clock due to different amounts of electrical power has been previously ascertained, this knowledge of the total retardation or acceleration upon the clock is, in fact, a record of the total amount of electricity energy which has been expended, or of the work done, since the last observation of the clock. The principle upon which the erg-meter is based is as follows: Let B C be the thick wire coil, and A B be the thin wire coil. If *c* be the current passing into the house and the difference of potential of supply and return cable, then *c v* is proportion al to the electric horse-power given to the house. Now the time of vibration of the pendulum is $t=\pi\sqrt{\frac{M}{g-kcv}}$. If the magnetic force is a retarding one M, g, v π are constants—that is, if the time of clock, when no electricity is passing, be called T, then $\frac{T}{t}=\sqrt{\frac{g-kcv}{g}}$. Now, care is taken that the magnetic forces are very small in comparison with gravitation forces acting on the pendulum bob; so that, to a degree of approximation which is sufficiently correct in practice, the above equation becomes $\frac{T}{t}=1-\frac{k}{2g}cv$. The rate of loss in the clock is therefore represented by $\frac{k}{2g}cv$. That is, it is proportional to c v, the electrical power. Hence, the total loss of the clock during any time represents the total electrical energy given to the house during that time. For the reasons stated in the earlier part of the paper, it is probable that the supply of electricity in a town will be carried out at a certain defined standard pressure of electro-motive force, which will be guaranteed to the consumer; and to assure the fulfillment of this condition, special steps will be taken by the local or other authority. The consumer need therefore but concern himself, as far as the supply he receives and has to pay for, with the quantity of electricity which he has made use of. In fact, the meter he makes use of to record it may be a coulomb meter. The work or energy meters will not be needed for general application, but only in special cases, where the complete record of the total electrical energy supplied is required. In what form, therefore, will it be most convenient for these coulomb meters to present their record? or, in other words, upon what unit should that register be based for commercial calculation? With the sole exception of those meters based upon electrolytic action, and which simply present the total of the increments of the metal, which are proportionate to the total quantity of current which is passed, the record presented is the result of a series of mechanical actions indicative of the strength and duration of the electrical supply. It is seen how these successive mechanical operations may be recorded in a cumulative way by the ordinary arrangement of a train of wheels; both in the ease of those meters based upon electrolytic action, and causing periodical reversals in the direction of the current, and also in those where the action due to the current is a purely mechanical one. There remains then but the question of the mode of graduation of the respective indicating dials. The unit already referred to, as being proposed to the Board of Trade in several provisional orders as "the energy in a current of 1000 ampères with an electromotive force of one volt flowing during one hour," has been shown to mean 1000 volt-ampere-hours, or 3,600,000 volt-coulombs. As it is probable that one of these factors—the volt—being fixed and constant, may be taken out of the commercial calculation for the consumer, it is worth while considering what dimensions the remaining factor—the coulomb—would assume under the ordinary conditions of practice; and hence what would be most convenient graduation of the dials to embrace these dimensions; but the commercial question requires also a financial expression in which to reckon these dimensions. The money value accompanying the suggestion of the above 1000 volt-ampere-hours unit, and which is sought to be attached primarily to 100 of these units is in some provisional orders 70s., in others 75s. Though it would be quite out of place in this paper to attempt to assign any financial value for the supply of electricity, yet for the nonce, and in carrying out the analysis of the above unit, as it might work under the conditions of actual practice, it is necessary to assume a money value. In the annexed table, which takes into consideration the effect of different rates of electro-motive force, according as they may be fixed, 70s. is assumed as the money for 100 units of 1000 volt-ampere-hours each.

Comparative Values of Supply of Electricity per 100 *units* (*of* 1000 *V.-A.-hours*)=£3 10*s.*

(1000 volt-ampere-hours=3,600,000 volt-coulombs.)

E.M.F. Volts.	Price.						
	100 Coulombs	10.000 Coulombs	100,000 Coulombs		1,000,000 Coulombs		
	Pence.	Pence.	s.	d.	£	s.	d.
100	.23	2.33	1	11.3	0	10	5
110	.26	2.57	2	1.7	1	1	5
120	.28	2.8	2	4	1	3	4
130	.3	3.03	2	6.3	1	5	3
140	.32	3.27	2	8.7	1	7	3
510	.35	3.5	2	11	1	9	2
160	.37	3 73	3	1.3	1	11	1
170	.4	3.97	3	0.7	1	13	1
180	.42	4.2	3	6	1	15	0
190	.44	4.43	3	8.3	1	16	11
200	.46	4 67	3	10.7	1	18	11

The prices in this table are based on the assumption that the price per coulomb unit remains at the same rate, irrespective of the intensity of the electromotive force. As a matter of fact, the amount of the latter ought certainly to be taken into account in fixing the sale price. Thus, with a pressure at 200 volts, the price for a certain number of coulombs ought not to exceed two-thirds of that of the same supply at 100 volts of pressure, in consequence of the economy in the former case, in the distributing mains, and in general working expenses. From a careful consideration of this table, it would appear that the amount of 100,000 coulombs would form a convenient unit of value whereby to reckon the commercial price. It would also serve as a basis of graduation for the indicating dials of the meters, with subdivision to $\frac{1}{10}$ and $\frac{1}{100}$ parts. The annexed diagram shows an arrangement of indicating dials, graduated in the first case with the unit suggested in the provisional orders, 1,000 ampere hours; in the second the unit is 100,000 coulombs as just suggested. The electro-motive force in each is taken at 100 volts. The proportion between the two systems of graduating in the present instance is as 3.6 to 10—3,600 being the number of coulombs which is the result of the flow of a current of one ampere for one hour; $\frac{3,600,000 \text{ coulombs}}{100 \text{ volts}}$ is, therefore, the actual quantity value of the first unit—1,000 ampere hours; as against 100,000 coulombs, the quantity contained in the second unit.

$$\begin{array}{l} \text{The values in} \\ \quad \text{C. G. S. units} \\ \quad \text{are} \end{array} \left\{ \begin{array}{l} \text{Volt-ampere-hour} \\ \quad \text{unit}=10^{10}. \\ \text{Coulomb unit } 10^{4}. \end{array} \right.$$

The advantage of the coulomb unit is that it expresses an actual quantity, whereas the ampere hour unit is but a time-rate, which as a matter of fact, has to be translated into its corresponding quantity. The coulomb method of graduation could be applied uniformly to all quantity meters, whereon they might happen to be afterwards used, inasmuch as the special rate of electro-motive force, or standard pressure, in each particular locality would be taken into account in the price there charged for the electric supply. This uniformity in manufacture would further reduce the expense of this, already the simpler form of registering meter. With the volt-ampere method, to use a quantity meter—as shown in the index-dial diagram—requires the exact position of the unit to be set according to the particular electro-motive force with which it is used, inasmuch as the quantity record is only one factor in the unit. This limited use of each particular recording meter would tend to create error and confusion. An energy meter alone would supply a complete record of the volt-ampere-hour mode of measurement; and this is a form of meter which involves a clock or some other time record, and consequently is a more expensive apparatus, and one requiring an amount of attention from the consumer which it would be impossible, in many cases, to expect. Thus, though the volt-ampere system may present, theoretically, a complete and convenient means of recording the supply of electricity, yet, in actual practice, the coulomb system would meet all wants, and be more suited to general application.

III.

ELECTRIC LIGHT ARITHMETIC.

BY

R. E. DAY, M. A.

ELECTRIC LIGHT ARITHMETIC.

ELECTRICAL RESISTANCE OF WIRES AND LAMPS.

THE electrical resistance (R) of a conductor varies directly with its length (L), and inversely as its sectional area (S), and may be expressed in terms of these quantities by the equation:

$$R=a\frac{L}{S},$$

where a is a constant quantity depending upon the material of the conductor, and is numerically equal to the resistance of a wire of this material whose length and cross section are each unity. It is called the *specific resistance* of the material.

Hence if we have two wires whose lengths are l_1 and l_2, their sectional areas s_1 and s_2, their specific resistances a_1 and

a_2, and their actual resistances R_1 and R_2 we shall have

$$R_1 = a_1 \frac{l_1}{s_1} \text{ and } R_2 = a_2 \frac{l_2}{s_2},$$

whence by division we get

$$\frac{R_2}{R_1} = \frac{a_2}{a_1} \times \frac{l_2}{l_1} \times \frac{s_1}{s_2};$$

and by means of this equation all problems connected with the relative resistances of wires in simple circuit can be readily solved.

Example 1. If the resistance of one mile of a certain electric light cable be 3.58 ohms, what is the resistance of 3.7 miles of the same cable?

If we apply the data of this question to the above formula we find that

$$\left.\begin{array}{l} s_1 = s_2 \\ a_1 = a_2 \\ l_1 = 1 \\ l_2 = 3.7 \\ R_1 = 3.58 \end{array}\right\} \begin{array}{l} \text{and } \therefore \text{ the equation becomes} \\ R_2 = 3.58 \times 3.7 = 13.25 \text{ ohms.} \end{array}$$

Example 2. If the resistance of 700 yards of a certain cable be 0.91 ohm, what will be the resistance of 1320 yards? *Ans.* 1.92 ohm.

Example 3. The resistance of 100 yards of a certain wire is equal to 5 ohms, what length of the same kind of wire would have a resistance of 13.2 ohms?

In this case the quantity whose value is required is l_2, and therefore transposing the equation (on page 1) we have

$$l_2 = l_1 \times \frac{R_2}{R_1} \times \frac{a_1}{a_2} \times \frac{s_2}{s_1}.$$

But in the present case $a_1 = a_2$, and $s_1 = s_2$, and the equation reduces to

$$l_2 = l_1 \times \frac{R_2}{R_1},$$

and from the data of the question $l_1 = 100$, $R_1 = 5$, $R_2 = 13.2$.

Therefore,

$$l_2 = 100 \times \frac{13.2}{5} = 263 \text{ yards.}$$

Example 4. The resistance of a certain cable is found to be 4.55 ohms, and the resistance of a mile of this cable is known to be 1.3 ohm. What is its length? *Ans.* 3.5 miles.

Example 5. If the resistance of 130 yards of copper wire 1-16th of an inch in diameter to be one ohm, what is the resistance of the same length of copper wire 1-32nd of an inch in diameter?

Since the areas of circles are proportional to the squares of their diameters, we have

$$\frac{s_1}{s_2}=\left(\frac{32}{16}\right)^2=4\,;$$

also,

$$\left.\begin{array}{r} a_2 = a_1 \\ {}_2 = l_1 \\ R_1 = 1 \end{array}\right\}$$

and therefore the fundamental formula reduces itself to

$$R_2=R_1\times\frac{s_1}{s_8}=4 \text{ ohms.}$$

Example 6. What is the resistance of a mile of copper wire which has a diameter of 65 mills, if the resistance of a mile of copper wire 80 mills in diameter is 8.29 ohms? *Ans.* 12.56 ohms.

N. B.—A *mill* is the one-thousandth part of an inch.

Example 7. What is the diameter of a copper wire a mile long which has a resistance of 23 ohms, if a mile of copper wire 70 mills in diameter has a resistance of 10.82 ohms?

Led d_1 and d_2 represent the diameters of the wires in mills, then we have

$$\frac{s_2}{s_1}=\left(\frac{d_2}{d_1}\right)^2$$

and

$$\frac{d_2}{d_1}\sqrt{\frac{s_2}{s_1}};$$

and from the data of the question

$$\frac{s_2}{s_1}=\frac{R_1}{R_2};$$

whence

$$d_2=d_1\sqrt{\frac{R_1}{R_2}}=70\sqrt{\frac{10.82}{23}}=48 \text{ mills.}$$

Example 8. A length of a thousand feet of wire 95 mills in diameter has a resistance of 1.15 ohm: what is the diameter of a wire of the same material of which the resistance of 1,000 feet is 10.09 ohms? *Ans.* 32 mills.

Example 9. Find the resistance in ohms of 500 yards of copper wire 165 mills in diameter, the resistance of one mile of pure copper wire 230 mills in diameter being equal to one ohm.

Since $$\frac{s_2}{s_1}=\left(\frac{d_2}{d_1}\right)^2$$

the fundamental formula becomes

$$R_2=R_1\times\frac{l_2}{l_1}\times\left(\frac{d_1}{d_2}\right)^2=\frac{500}{1760}\times\left(\frac{230}{165}\right)^2$$

$$=0.55 \text{ ohms.}$$

Example 10. If we take as our unit of electrical resistance that of a meter of pure copper wire one millimeter in diameter, what will be the number expressing the resistance of a copper wire 6,358 feet long and 0.0336 inch in diameter, assuming that one foot = 0.3048 of a meter? *Ans.* 2.66.

Example 11. A copper wire 6 meters long is found to have a diameter of 0.74 of a millimeter; what will be the length of a copper wire of one millimeter diameter which will offer the same electrical resistance?

Since $R_2 = R_1$, and $a_2 = a_1$, the fundamental equation becomes

$$l_2 = l_1 \times \left(\frac{d_2}{d_1}\right)^2 = 6 \times \left(\frac{1}{.74}\right)^2 = \begin{matrix}10.957 \\ \text{meters.}\end{matrix}$$

Example 12. What length of copper wire 4 millimeters in diameter would be equivalent to 12 yards of copper wire one millimeter in diameter?

Ans. 192 yards.

Example 13. Find the resistance of 15 miles of iron wire 0.3 inch in diameter having given that the resistance of one foot of iron wire 0.001 inch in diameter is 59.1 ohms.

Since $a_2 = a_1$ the fundamental formula becomes

$$R_2 = R_1 \times \frac{l_2}{l_1} \times \left(\frac{d_1}{d_2}\right. = 59.1 \times \frac{15 \times 1760 \times 3}{(300)^2}$$

$$= 52 \text{ ohms.}$$

Example 14. The resistance of 47 feet of copper wire 22 mills in diameter being 1 ohm, find the resistance of 200 yards of copper wire 134 mills in diameter?

Ans. 0.34 ohm.

Example 15. What must be the length of an iron wire the area of its cross section being four square millimeters, if it is to have the same electrical resistance as a wire of pure copper 1000 yards long, whose sectional area is one square millimeter, taking the conductivity of iron equal to 1-7th of that of copper?

Since in this case the value of l_2 is required, we transpose the fundamental formula thus

$$l_2=l_1\times\frac{R_2}{R_1}\times\frac{s_2}{s_1}\times\frac{a_1}{a_2},$$

and from the data of the question $R_2 = R_1$; $a_3 = 7a_1$; $s_2 = 4s_1$ and $l_1 = 1000$. Substituting these values we get

$$l_2\frac{1000\times4}{7}=571\tfrac{3}{7}\text{ yards.}$$

Example 16. How thick must an iron wire be so that for the same length it shall offer the same electrical resistance as a copper wire 2.5 millimeters in diameter? *Ans.* 6.61 millimeters.

Example 17. If the specific resistance per cubic centimeter of a certain metal be 13.36 microhms, what will be the resistance of a wire of this material one meter long and two millimeters in diameter?

N. B.—The microhm is equal to the millionth part of an ohm.

Referring to the definition of specific resistance given on page 1, we have in the present case

$$a_2=a_1\ ;\ l_2=100\ ;\ l_1=1\ ;\ s_1=1\ ;\ s_2=\frac{\pi}{100}\ ;$$

$$R_1=13.36,$$

and substituting these numerical values in fundamental equation

$$R_2=\frac{13.36\times100\times100}{\pi}=42526 \text{ michroms.}$$

Example 18. A wire one foot long and one mill in diameter has a resistance of 9.15 ohms: find the specific resistance per cubic inch of the material of this wire. *Ans.* 0.5989 michrom.

Example 19. Find the resistance of 20 yards of platinum wire 0.016 inch in diameter, the relative resistance of platinum with respect to copper being 11.3.

Referring to example 14 we find that the resistance of 200 yards of copper wire 134 mills in diameter is 0.34 ohm, and therefore for the quantities in the fundamental formula we have

$$R_2 = .34 \times \frac{20}{200} \times \left(\frac{134}{16}\right)^2 \times \frac{11.3}{1} = \frac{26.95}{\text{ohms}}$$

Example 20. What would be the resistance of 7 miles of iron wire 238 mills in diameter, the relative resistance of this particular iron with respect to copper being 7.5 ? *Ans.* 49.79 ohms.

Example 21. What should be the respective lengths of two wires of silver and lead so that they may each offer the same resistance as 10 inches of copper wire of the same thickness, the conductivity of silver and lead with respect to copper being 1.0467 and 0.0923 respectively?

Ans. 10.467 and 0.923 inch.

Example 22. What must be the relative thicknesses of wires of iron, silver and platinum, of the same length, so that their resistances may be equal?

Ans. As 100 : 36 : 122.

Example 23. If the resistance of a wire 3 metres long and weighing 3 grammes be 5.88 ohms, what is the specific resistance per cubic centimetre of the material, its specific gravity being 20.337 ?

If s_1 represent the sectional area of the wire expressed in square centimetres, then we have

$$s_1 \times 300 \times 20.337 = 3$$

$$\therefore \qquad s_1 = \frac{3}{300 \times 2033.7}$$

of a square centimetre ; also $l_1 = 300$; $s_2 = 1$ and $a_2 = a_1$; and therefore the fundamental formula becomes

$$R_2 = 5.88 \times \frac{1}{300} \times \frac{3}{300 \times 20.337}$$

$$= \frac{5.88}{300 \times 2033.7} \text{ ohm} = 9.638 \text{ microhms.}$$

Example 24. Determine the specific resistance per cubic millimetre of a wire 437 millimetres long, which has a resistance of 0.1257 ohm, and which weighs 0.411 gramme in air, and 0.365 gramme in water.

The weight of water displaced = .046

gramme, and therefore the specific gravity of the material

$$=\frac{411}{46}=8.935,$$

and then following the method of the last example we get R=30.278 microhms.

Example 25. Find the specific resistance per *cubic centimetre* of the material of which the above wire is composed.

Ans. 3.0278 microhms.

Example 26. A wire 874 millimetres long and weighing 0.822 gramme in vacuo and 0.73 gramme in water, has a resistance of 0.1257 ohm; find the specific resistance per cubic millimetre of the material. *Ans.* 15.14 microhms.

Example 27. A piece of silver wire 3 feet long was found to weigh 7.2 grains, and its resistance was 0.3026 ohm. What would be the resistance of a wire one foot long weighing one grain?

If w_1 and w_2 be the weights of two wires of the same material whose lengths are l_1 and l_2 and their sectional areas s_1 and s_2, then

$$\frac{w_2}{w_1}=\frac{l_2 s_2}{l_1 s_1}\text{ and }\therefore\frac{s_1}{s_2}=\frac{l_2}{l_1}\times\frac{w_1}{w_2}$$

Substituting this value for $\frac{s_1}{s_2}$ in the fundamental formula, and remembering that $a_1=a_2$, we have

$$R_2=R_1\times\left(\frac{l_2}{l_1}\right)^2\times\frac{w_1}{w_2}$$

$$=.3026\times\left(\frac{1}{3}\right)^2\times\frac{7.2}{1}=0.24\text{ ohm.}$$

Example 28. What are the relative resistances of two wires, one of which is 30.48 centimetres long and weighs 35 grammes, while the other is 18.29 centimetres long and weighs 10.5 grammes?

Ans. As 12 : 10, nearly.

Example 29. Find the resistance at 0° C. of 20 metres of German silver wire weighing 52.5 grammes, having given that the resistance at 0° C. of a wire of this material 1 metre long and weighing 1 gramme is 1.85 ohm.

Ans. 14.1 ohms.

Example 30. If the resistance of a foot of pure copper wire weighing one grain be 0.2106 ohm, and the resistance of a piece of ordinary copper wire 3 feet long and weighing 3.45 grains was found to be 0.5782 ohm; compare the conducting power of this sample of wire with that of a similar one of pure copper.

If the second wire were of pure copper, then by the method adopted in the solution of Example 27 we should find that its resistance

$$R=0.2106\times\left(\frac{3}{1}\right)^2\times\frac{1}{3.45}=0.5494\,\text{ohm.}$$

Therefore

$$\frac{\text{Conducting power of this wire}}{\text{“ \quad “ \quad “ a similar one of pure copper}}$$

$$=\frac{5494}{5782}=\frac{95}{100}\text{nearly.}$$

Hence it appears that the *conductivity* of this specimen of copper wire is about 95 per cent. of that of pure copper.

INTENSITY OF CURRENT IN A SIMPLE CIRCUIT.

All problems connected with this part of the subject are worked out by the aid of a formula which is known as Ohm's law; namely

$$I=\frac{E}{R}$$

in which I represents the intensity of the current; E the resultant of all the electro-motive forces in circuit; and R the total resistance of the circuit.

Example 1. The internal resistance of a certain Brush dynamo machine is 10.9 ohms, and the external resistance is 73 ohms; the electro-motive force of the machine being 839 volts. Find the strength of the current flowing in the circuit.

In this case we have

$$E=839\,;\ R=73+10.9=83.9 \text{ ohms,}$$

and therefore substituting these values in the above equation we get

$$I=\frac{839}{83.9}=10 \text{ amperes.}$$

Example 2. The resistance of the dynamo machine being 1.6 ohm, and the external resistance 25.4 ohms; calculate the strength of current in the circuit, the electro-motive force being equal to 206 volts. *Ans.* 7.6 amperes.

Example 3. Three arc lamps in series have a resistance of 9.36 ohms, while the resistance of the leading wires is 1.1 ohm, and that of the dynamo machine is 2.8 ohms. Find what must be the electro-motive force of the machine when the strength of the current produced is 14.8 amperes.

In this case we have

$$R = 2.8 + 9.36 + 1.1 = 13.26 \text{ ohms},$$
$$I = 14.8 \text{ amperes};$$

and therefore substituting these values in the equation

$$E = I \times R$$

it becomes

$$E = 13.26 \times 14.8 = 196.3 \text{ volts}.$$

Example 4. A certain arc lamp has an electrical resistance of 2.5 ohms, while that of the leading wires is .5 ohm, and that of the machine 0.5 ohm. What must be the electro-motive force of the machine so as to send a current of 25 amperes through three of these lamps and the leading wires?

Ans. 213 volts nearly.

Example 5. Calculate from the following data the average resistance of each of three arc lamps which are arranged in series. The electro-motive force of the machine is 244 volts and its resistance is 3.7 ohms, while that of the leading wires is 2 ohms, and the strength of current through each lamp is 21 amperes.

If x represent the average resistance in ohms of each lamp, then the total resistance of the circuit is

$$R=3x+2+3.7.$$

But by Ohm's law

$$R=\frac{E}{I}$$

$$\therefore 3x+5.7=\frac{244}{21}=11.61 \text{ ohms},$$

$$\therefore x=1.97 \text{ ohms nearly.}$$

Example 6. In an experiment with a Gramme machine working on a simple circuit it was found that the electromotive force developed by the machine was 81.58 volts, the strength of current 29.67 amperes, and the external resistance 1.14 ohm. What was the external resistance of this Gramme machine?

Ans. 1.61 ohm.

Example 7. In a similar experiment with another dynamo machine its resistance was known to be equal to 4.58 ohms, while the electro-motive force was 158.5 volts, and the current 17.5 amperes. What was the resistance of the exterior part of the circuit? *Ans.* 4.48 ohms.

Example 8. Three Maxim incandescent lamps were placed in series. The average resistance, when hot, of each lamp was 39.3 ohms, and that of the dynamo machine and leading wires 11.2 ohms. What electro-motive force was required to maintain a current of 1.2 ampere through the circuit?

In this case we have

$$R=3\times 39.3+11.2=159.1 \text{ ohms}$$

and $I=1.2$ ampere;

and therefore by Ohm's law

$$E=I\times R=1.2\times 129.1=154.9 \text{ volts.}$$

Example 9. Two incandescent lamps, each having a hot resistance of 73 ohms, were joined up in series to the poles of a dynamo machine, and the total resistance of the circuit was 158 ohms. What was the electro-motive force of the machine when a current of 1 ampere was circulating through it? *Ans.* 158 volts.

Example 10. The resistance of the arc of a certain Brush lamp was 3.8 ohms when a current of 10 amperes was flowing through it. What was the electro-motive force between the two terminals?

By Ohm's law

$$E = I \times R = 10 \times 3.8 = 38 \text{ volts.}$$

Example 11. The electro-motive force between the terminals of a Serrin lamp was 31.1 volts, and there was a current of 35.8 amperes flowing through it. What was the resistance of the arc?

Ans. 0.869 ohm.

Example 12. Twenty-five exactly similar galvanic cells, each of which had an average internal resistance of 15 ohms, were joined up in series to one incandescent lamp of 70 ohms resistance, and produced a current of 0.112 amperes. What would be the strength of current produced by a series of 30 such cells through 2 lamps, each of 30 ohms resistance?

The data of the first part of the problem enable us to determine the average electro-motive force of each cell of the battery. Let this be represented by E, then we have

$$25\ E = I \times R = .112 \times (25 \times 15 + 70)$$
$$= .112 \times 445$$

$$\therefore\ E = \frac{.112 \times 445}{25} = 2 \text{ volts nearly.}$$

Then from the data in the second part of the problem we have by Ohm's law

$$I = \frac{30 \times 2}{30 \times 15 + 2 \times 30} = \frac{60}{510} = 0.118 \text{ amperes.}$$

HEATING EFFECT OF THE CURRENT IN A SIMPLE CIRCUIT.

Whenever an electrical current flows through a conductor heat is invariably produced, and Joule has shown experimentally that the total quantity of heat developed in a circuit may be expressed by the formulæ

$$Q=I^2\times R=\frac{E^2}{R}=E\times I,$$

where Q is the quantity of heat produced, and I and R have the same meanings as before.

So also if we consider any particular portion of a circuit the resistance of which is r, and where the difference of potential at the two ends of this section is e, then the quantity of heat developed in this portion of the circuit in the unit of time may be expressed by the formula

$$q=I^2r=\frac{e^2}{r}=eI.$$

The mechanical equivalent of the quantity of heat developed in t units of time in a circuit, the resistance of which is R by a current of strength I, is given by the equation

$$W=JQ=I^2Rt,$$

where J is Joule's dynamical equivalent of heat and is equal to 4.2×10^7 ergs in the C.G.S. system of units.

Example 1. One portion of an electrical circuit was composed of 10 feet of copper wire 7 mills in diameter, and another portion of 5 feet of iron wire 148 mills in diameter. If the resistances of copper and iron are to one another as 112 to 825, compare the heating effects of the current in these two portions of the circuit.

Since the same current flows through both wires the value of I is the same for both, and for their relative resistances we have

$$\frac{R_2}{R_1}=\frac{a_2}{a_1}\times\frac{l_2}{l_1}\times\left(\frac{d_1}{d_2}\right)^2=\frac{825}{112}\times\frac{5}{10}\times\left(\frac{7}{148}\right)^2$$

$$=\frac{1}{121}\text{ nearly.}$$

Therefore

$$\frac{\text{Heat developed in the copper section}}{\text{“ \quad “ \quad “ \quad iron \quad “}}$$

$$=\frac{\text{Resistance of copper}}{\text{“ \quad “ iron}}=\frac{121}{1}$$

Example 2. The interpolar portion of an electrical circuit consisted of two different wires joined end to end. One was of copper 3 feet long and 65 mills in diameter, while the other was of platinum 6 inches long and 35 mills in diameter. The resistances of copper and platinum being to one another as 112 to 1243, compare the quantities of heat developed by the current in these portions of the circuit.

$$\textit{Ans.}\ \frac{\text{Heat in platinum}}{\text{" " copper}}=\frac{638}{100}$$

Example 3. The internal resistance of a certain dynamo machine is .5 ohms, that of the lamp is 10 ohms, and of the leading wires 2 ohms. Compare the quantities of heat produced in the lamp and in the machine, and find what proportion of the total energy in the circuit will appear in the lamp.

$$\textit{Ans.}\ \frac{\text{Heat in lamp}}{\text{" " machine}}=\frac{10}{5}=\frac{2}{1},$$

$$\frac{\text{Energy in lamp}}{\text{Total energy in circuit}}$$

$$=\frac{\text{Resistance of lamp}}{\text{Resistance of whole circuit}}$$

$$=\frac{10}{5+2+10}=\frac{10}{17}.$$

Example 4. The Swan incandescent lamps, each of which has a resistance of 35 ohms, are connected in series by thick wires with a series of 40 Grove cells, the average resistance of each cell being 0.5 ohm. Compare the quantity of heat developed in each lamp with that developed in each cell of the battery, and find what proportion of the total energy yielded by the battery appears in each lamp:

$$\textit{Ans.}\ \frac{\text{Heat in lamp}}{\text{`` `` cell}}=\frac{70}{1};$$

$$\frac{\text{Energy in one lamp}}{\text{`` of battery}}=\frac{7}{18}$$

Example 5. The internal resistance of a voltaic battery is equal to that of 3 metres of a particular wire. Compare the quantities of heat produced both inside and outside the battery, when its poles are connected by one metre of this wire, with the quantities produced in the same time when they are connected by 37 metres of the same wire.

Taking the resistance of one metre of the given wire as our unit and I_1, I_2 as the intensities of the current in the two cases, we have

$$I_1=\frac{E}{4}\text{ and } I_2=\frac{E}{40};\text{ whence }\frac{I_1}{I_2}=10,$$

and therefore for the heat developed in the battery in the two cases

$$\frac{Q_1}{Q_2}=\frac{I_1{}^2R_1}{I_2{}^2R_2}=\left(\frac{I_1}{I_2}\right)^2\frac{100}{1}\text{ because } R_1=R_2$$

For the heating effects outside the battery

$$\frac{Q_1}{Q_2}=\left(\frac{I_1}{I_2}\right)^2\frac{R_1}{R_2}=\frac{100}{1}\times\frac{1}{37}=\frac{100}{37}.$$

Example 6. A Grove cell whose electromotive force is 1.9 volt and internal resistance 0.4 ohm has its poles connected (1) by a wire of three ohms, and (2) by a wire of 30 ohms resistance. Compare the amounts of heat developed in the cell in the two cases. *Ans.* As 80 : 1.

Example 7. A dynamo machine the resistance of which is 6 ohms has its poles connected (1) by a circuit of .5 ohm resistance, and (2) by a circuit of 500 ohms resistance. Compare the quantities of energy which are dissipated as heat in the machine assuming that the driving engine works always at the same horse-power. *Ans.* As 78 : 1 nearly.

Example 8. The interpolar portion of a circuit consisted of a piece of copper and a piece of platinum wire. The platinum was 1 foot long and 28 mills in diameter, while the copper was 20 yards long and 65 mills in diameter. Compare the quantities of heat developed, per running foot, in each of these wires.

Ans. As 60 : 1 nearly.

Example 9. The difference of potential between the ends of the two carbons of a certain Brush arc light is 36 volts, and a current of 10 amperes is passing through it. Find the quantity of heat developed per second.

When expressed in the Centimetre-Gramme-Second (C.G.S.) system of units.

One volt $=10^8$ C.G.S. units of potential,

One ohm $=10^9$ C.G.S. electro-magnetic units of resistance,

One ampere $=10^{-1}$ of the C.G.S. unit of current.

One water-gramme degree Centigrade $=4.2\times10^7$ ergs.

In the present case we have

$$W=E\times I=36\times10^8\times10\times10^{-1}$$
$$=36\times10^8 \text{ ergs per second,}$$

$$\therefore Q=\frac{36\times10^8}{4.2\times10^7}=\frac{360}{4.2}=85.72 \text{ w.g.d. C}^\circ.$$

Example 10. An Edison incandescent lamp has a resistance of 125 ohms, and the difference of potential between the ends of the carbon filament is 110 volts. Find the strength of current flowing through it and the quantity of heat developed in the lamp per second.

Ans. Current = 0.88 ampere. Heat = 23.05 w.g.d. C°.

Example 11. The house main wire for supplying 20 Edison lamps, each of which requires a current of 0.8 amperes is of copper of 86 per cent. conductivity, and 65 mills in diameter. What is the amount of heat developed per second in every foot of this wire?

We shall find that the resistance of

one foot of this wire is 0.00274 ohm, and then substituting the numerical values in the equation

$$Q = R \times I^2$$

$$= 2.74 \times 10^{-3} \times 10^9 \times (.08 \times 20)^2 \text{ ergs per second.}$$

$$= \frac{2.74 \times 10^6 \times (1.6)^2}{4.2 \times 10^7} = 0.167 \text{ w.g.d. C}^\circ.$$

Example 12. A current of one ampere circulates through a wire whose resistance is 0.9536 ohm. Find the amount of heat developed in this wire in 5 minutes.

Ans. 68 w.g.d. C°.

Example 13. A current of 0.75 ampere was sent for 5 minutes through a column of mercury whose resistance was 0.47 ohm. The mass of the mercury was 20.25 grammes and its specific heat 0.0332; find the rise of temperature, assuming that no heat escapes by radiation.

By the method adopted in the previous examples we find that the number of units of heat developed in 5 minutes by the passage of a current of 0.75 ampere through a resistance of 0.47 ohm is 18.88 units, and by the question this quantity of heat will raise 20.25 grammes of mercury through x° C. of temperature.

But the amount required for this purpose is

$$=20.25\times 0.0332\times x$$
$$=18.88 \text{ units by the question.}$$

Hence

$$x=\frac{18.88}{20.25\times 0.0332}=28^\circ \text{ C.}$$

Example 14. A coil of very fine wire which had a resistance of 46.64 ohms was placed in 1000 grammes of water at o° C. The current from a series of 50 voltaic cells, each of which had an electro-motive force of 1 volt and a resistance of 6 ohms, was sent through the coil for 10 minutes. Find the rise of temperature of the water.

Ans. 1.39 C.

Example 15. A small coil of German silver wire was placed in one of Lenz's alcohol calorimeters, and a steady current of 1.8 ampere was sent through it. The resistance of the coil was 1.2 ohm, and it was found that the temperature of the alcohol rose 1° C. in 23 seconds. Had the resistance of the coil been equal to 1 ohm while the current strength was the same, what would have been the time required?

Ans. 27.6 seconds.

Example 16. A wire of pure copper, 0.165 centimetre in diameter, has a current of 10 amperes flowing through it. What will be the limiting temperature of the wire?

The specific resistance of pure copper per cubic centimetre is 1.65 microhm, and therefore the resistance of one metre of the above wire will be 7.8×10^{-3} ohm. Therefore the quantity of heat developed per second in one metre of this wire is

$$= \frac{7.8 \times 10^{6}}{4.2 \times 10^{7}} = .1857 \text{ thermal unit.}$$

The surface of a length of one metre of this wire is

$$= 100 \times \pi \times 0.165 = 51.8 \text{ square centimeters.}$$

Hence the development of heat in this wire is at the rate of $\frac{.1857}{52.2}$ or 0.00356 of a thermal unit per square centimetre of surface per second. But the rate of loss of heat by radiation and convection from an unpolished surface of copper (or other solid material) is about 1-4000th per square centimetre per degree of excess of temperature above that of the surrounding medium. It follows therefore that the temperature of this wire will go on rising until it has reached such a value that the loss per second by radiation and convection is equal to 0.00356 of a thermal unit. Hence if x° be this temperature

$$\frac{x}{4000}=.00356 \text{ and } \therefore\ x=14^\circ.32 \text{ C.}$$

Hence, if this wire be freely exposed to the air, its limiting temperature will be 14°.24 C. above that of the surrounding air.

Example 17. Taking account of the loss of heat by radiation, calculate what would be the rise of temperature of the wire referred to in Example 11.

Ans. 42°.3 C.

Example 18. A short piece of lead wire is included as a *safety catch* in a circuit, and it is required to find what its diameter should be so that a current of 7.2 amperes may just fuse it, assuming that the specific resistance of lead per cubic centimetre is 19.85 microhms, and that the melting point of lead is 335° C.

Let $x=$ diameter of the wire in centimetres. The resistance of a lead wire one centimetre long and x centimetres in diameter is

$$=19.85\times10^{-6}\times\frac{4}{\pi x^2}\text{ ohms,}$$

and therefore the quantity of heat developed in one second in one centimetre of this wire

$$=\frac{(.72)^2\times19.85\times10^3\times4}{\pi x^2\times4.2\times10^7}\text{ w.g.d.C°.}$$

But the area of the surface of one centimetre of this wire is πx square centimetres, and therefore the development of heat per square centimetre of surface is at the rate of

$$\frac{(.72)^2\times19.85\times10^3\times4}{\pi^2x^3\times4.2\times10^7}\text{thermal units per}$$

second. Now while the current is flowing the wire will go on rising in temperature,. but the rate of increase will be slower and slower as the temperature approaches the limiting one at which the gain of heat per second is just balanced by the loss by radiation and convection, and if this limiting temperature be equal to or higher than that at which lead fuses, then the wire will melt and the current will be interrupted. If therefore the diameter of the wire is to be such that the wire shall *just* melt with the above current we have the equation

$$\frac{335}{4000}=\frac{(.72)^2\times 19.85\times 10^3\times 4}{\pi^2 x^3\times 4.2\times 10^7}$$

whence

$$x^3=\frac{4000\times(.72)^2\times 19.85\times 10^3\times 4}{335\times\pi^2\times 4.2\times 10^7}$$

and

$\therefore x=.106$ centimetre, or about No. 19 B.W.G.

For the sake of simplicity it has been assumed in this solution that the resistance of the lead does not vary with the temperature, and also that the rate of loss of heat by radiation follows the same law at comparatively high temperatures. The student should also notice that the value of the *specific heat* of lead only affects the *time* which would elapse from the commencement of the flow before the wire fuses, and does not affect the practical question as to whether or not fusion will ultimately take place.

Example 19. A leaden *safety catch* is to be inserted in a circuit which will fuse if the current strength exceeds 20 amperes. What must be its diameter?

N.B.—A *safety catch* is a short piece of wire whose thickness and material are so chosen that if it be inserted in a circuit it will fuse and so interrupt the circuit if the strength of the circuit exceeds a given value. In the present case we shall find by the method adopted in Example 18 that $x = .209$ centimetre, or the wire is about No. 14 B.W.G.

Example 20. A copper wire is to be inserted in a circuit as a safety catch for a current of 500 amperes. Taking the melting point of copper at 1050° C. and its specific resistance per cubic centimetre equal to 1.652 microhm; find what must be its diameter.

Ans. 1.53 centimetre, or about No. 10 B.W.G.

WORK UTILIZED IN A SIMPLE CIRCUIT.

Example 1. The difference of potential between the electrodes of a certain Swan lamp was 31.1 volts, and the current flowing through it was 1.22 ampere. Calculate the energy, in horse-power, absorbed by the lamp, and also the amount of heat developed.

By Joule's formula we have

$$W=I^2R=I\times E,$$

and expressing the values of the volt and ampere in absolute C. G. S. measure we have in the present case

$$W=\frac{1.22}{10}\times 31.1\times 10^8 \text{ ergs}$$

$$=3.7942\times 10^8 \text{ ergs per second.}$$

But one-horse power is equivalent to 7.46×10^9 ergs, and therefore the energy absorbed by the lamp per second expended appears in the lamps, find the amount of horse-power required to maintain a current of 10 amperes.

Ans. 15.73 H.P.

Example 8. A certain size of Swan lamp has, when hot, a resistance of 30 ohms. There are 30 of these lamps in series, and the resistance of the remainder of the circuit is 15 ohms. What horse-power will be required to maintain a current of 1.5 ampere through this series of lamps?

Ans. 2.76 H.P.

Example 9. An electric circuit is composed of 5 arc lamps, each of which has a resistance of 2.5 ohms, the armature of the machine which has a resistance of 2 ohms, and the leading wires which have a resistance of 1.5 ohm. Find what proportion of the total energy expended in the circuit appears in the lamp.

The total resistance of the circuit = 16 ohms, and the ratio of the work expended in any part of the circuit to the total work is equal to the ratio of the resistance of that part to the resistance of the whole circuit. Hence in the present case the proportion of the total energy which is developed in the lamps

$$=\frac{12.5}{16}=.78 \text{ or } 78 \text{ per cent.}$$

Example 10. If the internal resistance of a dynamo machine be 3 ohms, and that of the external circuit 18 ohms: what is the ratio of the external or useful work of the current to the total work expended?

Ans. 6–7ths.

Example 11. The resistances of two incandescent lamps arranged in series are 50 and 100 ohms respectively, and the total resistance of the circuit is 200 ohms. What fraction of the total work is expended in each of these lamps?

Ans. $\frac{1}{4}$ and $\frac{1}{2}$.

Example 12. In an experiment with a certain dynamo machine the resistance of the armature was 0.016 ohm, and that of the external circuit 0.757 ohm. The power required to turn the armature at a certain rate in the magnetic field was 7.604 H.P. and this produced a current of 83.7 amperes. Calculate the *duty* of the generator and also its *commercial efficiency*.

The total resistance of the circuit $= 0.757+0.016=0.773$ ohm, and therefore by the method of Example 3 we find that the electrical energy in the current

$$=\frac{(8.37)^2\times 0.773}{7.46}=7.259 \text{ H.P.}$$

The *duty* of the generator is the ratio of the total electrical energy developed to the amount of energy expended in turning the armature in the magnetic field. In the present case it is

$$=\frac{7.259}{7.604}=0.955.$$

The *commercial efficiency* of the generator is the ratio of the amount of electrical energy which appears in the external circuit to the total energy which is extended in driving the machine. In the present case the commercial efficiency

$$=\frac{757\times 7.259}{773\times 7.604}=0.935.$$

Example 13. The internal resistance of a certain dynamo machine was 1.5 ohm, and the external resistance 2.7 ohms. An expenditure of 3.87 H.P. in turning the armature developed a current of 25.5 amperes. Calculate from these data the *duty* of the generator and also its *commercial efficiency.*

Ans. Duty=0.95 ; Commercial efficiency =161.

Example 14. In an experiment with a Brush machine the work expended in driving the armature was 15.3 H.P., the internal resistance was 10.5 ohms, the external resistance 73 ohms, and the strength of the current was 10 amperes. Find the *commercial efficiency* of this machine. *Ans.* 0.64.

COMPOUND ELECTRICAL CIRCUITS.

Example 1. Two points in an electrical circuit are connected by two incandescent lamps, whose resistances are 31 and 37 ohms respectively. Find the resultant resistance of the circuit between these two points.

The conductivity of the two branches is

$$\frac{1}{31}+\frac{1}{37}=\frac{68}{31\times37},$$

and therefore their resultant resistance, which is the reciprocal of their conductivity, is

$$=\frac{31\times37}{68}=16.87 \text{ ohms.}$$

Example 2. Three conducting wires, whose resistances are 5, 7, and 9 ohms, are joined together in multiple arc. Find the resultant resistance of this compound conductor. *Ans.* 2.2 ohms.

Example 3. Four wires are joined together in multiple arc, their resistances being 5.5, 18, 3.7, and 2.9 ohms respectively. Find the resultant resistance of the compound conductor thus formed.

Ans. 1.17 ohms.

Example 4. An iron wire 20 metres long and 0.2 centimetres in diameter, a copper wire 105 metres long and 0.15 centimetre in diameter, and a German silver wire 0.6 metre long and 0.04 centimetre in diameter were joined together in multiple arc. Find the resultant resistance of the compound conductor thus formed, having given that

MICROHMS.

Specific resistance of iron
per cubic centimetre=9.825

Specific resistance of copper
per cubic centimetre=1.652

Specific resistance of German silver
per cubic centimetre=21.170

Ans. 0.277 ohm.

Example 5. The poles of a dynamo machine are connected in multiple arc by 20 incandescent lamps, each of which has a resistance of 80 ohms. The internal resistance of the machine being 0.25 ohm, what must be its electro-motive force so that a current of 1.2 amperes may flow through each lamp?

The resistance of the interpolar circuit
$=\frac{80}{20}=4$ ohms.

The resistance of the machine=0.25 ohm.

∴ total resistance of the circuit
=4.25 ohms.

The current flowing through the machine
$=20\times 1.2=24$ amperes,

and therefore for the value of the electromotive force in the circuit we have by Ohm's law

$$E=I\times R=24\times 4.25=102 \text{ volts.}$$

Example 6. Under the circumstances described in the last Example, what would be the electro-motive force in the outer circuit between the terminals of the machine?

This can readily be calculated from the known values of the resistance of any lamp and the strength of current flowing through it.

By Ohm's law

$$E=I\times R=1.2\times 80=96 \text{ volts.}$$

Example 7. A thousand incandescent lamps are joined up in simple multiple arc to the poles of a dynamo machine whose resistance is 0.004 ohm. If the average resistance of each lamp is 120 ohms, and it requires a current of 0.9 amperes to vitalize it, what must be the electro-motive force of the machine?

Ans. 111.6 volts.

Example 8. The internal resistance of a dynamo machine is 0.008 ohm, and it is employed to drive a current of 0.8 ampere through each of 900 incandescent lamps arranged in simple multiple arc. The resistance of each lamp when hot being 130 ohms, what must be the electromotive force of the machine?

Ans. 109.4 volts.

Example 9. The electro-motive force of a certain dynamo machine is 45 volts, and its resistance is 0.01 ohm. The lamps to be used with it have a hot resistance of 35 ohms. What number may be arranged in simple multiple arc so that a current of 1.2 ampere may flow through each of them?

If $x=$ number of lamps, then the resultant resistance of the external circuit is $\frac{35}{x}$ ohms, and the total resistance of the circuit is $\left(\frac{35}{x}+.01\right)$ ohms. Also the current flowing through the dynamo is $x \times 1.2$ ampere, and using Ohm's equation

$$E=I \times R,$$

we have

$$45 = x \times 1.2\left(\frac{35}{x} + .01\right)$$
$$= 1.2 \times 35 + x \times .012$$

and $\therefore\ x = 250$ lamps.

Example 10. Find the number of lamps which can be driven in simple multiple arc by a dynamo machine whose resistance is 0.032 ohm, and its electromotive force 55 volts, if each lamp has a hot resistance of 28 ohms and requires a current of 1.6 ampere.

Ans. 199 lamps.

Example 11. The internal resistance of a certain dynamo machine is 1 ohm, and its electro-motive force is 484 volts when the interpolar portion of the circuit consists of 200 incandescent lamps arranged in 20 series of 10 each. If the resistance of each lamp be 30 ohms, what is the strength of current flowing through it?

The resistance of each series of lamps

$$= 30 \times 10 = 300 \text{ ohms.}$$

The resultant resistance of the 20 series

$$= \tfrac{300}{20} = 15 \text{ ohms.}$$

Total resistance of the whole circuit

$$=15+1=16 \text{ ohms.}$$

Hence the current flowing through the machine

$$=\tfrac{484}{16}=30.25 \text{ amperes,}$$

and therefore the strength of the current through each one of the 20 series is

$$=\frac{30.25}{20}=1.51 \text{ ampere.}$$

Example 12. Thirty-five incandescent lamps, each of which has a hot resistance of 35 ohms, are arranged in 5 series of 7 each, and are then connected in multiple arc with the poles of a dynamo machine whose internal resistance is 2.5 ohms and its electro-motive force 309 volts. What is the strength of current through each lamp? *Ans.* 12 ampere.

Example 13. The resistance of a certain dynamo machine is 2 ohms, and it is required to arrange 80 incandescent lamps, each of which has a hot resistance of 50 ohms, in such a way that the resistance in the external circuit shall be 20 times that of the machine. How is this to be done?

Let x = number of series. Then $\frac{80}{x}$=number of lamps in each series, and $\frac{80\times 50}{x^2}$=resistance of the external circuit,

=40 ohms, by the conditions of the problem.

Hence,

$$x^2=\frac{80\times 50}{40}=100\,;\ \therefore\ x=10 \text{ rows.}$$

Example 14. A set of 120 incandescent lamps, each of 30.7 ohms resistance, are to be arranged in such a way that their resultant resistances shall be about 16 times as great as that of the dynamo machine, whose resistances is 1.6 ohm. How is this to be done ?

Ans. 12 series of 10 each.

DISTRIBUTION OF ENERGY IN A COMPOUND CIRCUIT.

Example 1. Two coils of platinum wire, whose resistances are as 5 : 3, are placed in vessels of water forming calorimeters, and are connected in series in an electric circuit, when it is found that they quantities of heat produced in 10 minutes are 72.22 and 43.33 units respectively. The coils are then placed in multiple arc and the quantities of heat produced in 10 minutes are then found to be 2.6 and 4.33 units respectively. Deduce from these experiments Joule's law as to the heating effect of an electric current.

(1) When the coils are in series.

Let H_1 represent the heat developed in 1st coil, and R_1 its resistance.

Let H_2 represent the heat developed in 2nd coil, and R_2 its resistance.

Then we have

$$\frac{H_1}{H_2}=\frac{72.22}{43.33}\text{ and }\frac{R_1}{R_2}=\frac{5}{3},$$

whence

$$\frac{R_1H_2}{R_2H_1}=\frac{5\times 43.33}{3\times 72.22}=1\ ;$$

$$\therefore \frac{H_2}{H_1}=\frac{R_2}{R_1}\quad . \ . \ . \ . \quad (a).$$

(2) When the coils are arranged in multiple arc.

In this case the current will divide itself between the two coils in the inverse ratio of their resistances, and therefore the current in the first coil will be 3-8ths, and that through the second coil 5-8ths of the main current.

Let I_1 and I_2 represent the strengths of the currents in the two coils, then we shall have

$$\frac{I_1{}^2 \times R_1}{I_2{}^2 \times R_2} \times \frac{H_2}{H_1} = \frac{9 \times 5 \times 4.33}{25 \times 3 \times 2.6}$$

$$= \frac{12.99}{13.00} = 1, \text{ very nearly.}$$

Therefore

$$\frac{H_2}{H_1} = \frac{I_2{}^2 \times R_2}{I_1{}^2 \times R_1} \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

and from (α) and (β) we get Joule's equation—

$H = KRI^2$ where K is some constant.

Example 2. A current of 2.4 amperes is sent through two lamps in series, each of which has a resistance of 60 ohms. The lamps are then placed in multiple arc and the same current is divided between them. Compare the quantities of heat developed in each lamp in the two cases.

Ans. As 4 : 1.

Example 3. The resistance of a certain dynamo machine is 2 ohms, and when it is making 820 revolutions a minute it can drive a current of 2.5 amperes through a single extrapolar resistance of 30 ohms. The extrapolar part is then made up of 5 incandescent lamps, placed in multiple arc, and each having a resistance of 150 ohms. Assuming that the electro-motive force is proportional to the velocity of rotation, find the rate at which the armature must be made to turn so as to send a current of 1 ampere through each of these lamps.

Under the conditions stated in the first part of the problem, the electro-motive is found directly by Ohm's law—

$$E_1=I_1\times R_1=2.5\times(2+30)=80 \text{ volts.}$$

In the second arrangement the resultant resistance of the circuit is

$$R_2=2=\tfrac{150}{5}=32 \text{ ohms,}$$

and the current flowing through the dynamo is equal to 5 amperes, hence its electro-motive force is

$$E_9=5\times32=160 \text{ volts;}$$

and since the electro-motive force is proportional to the velocity of rotation, the velocity required is

$$=\frac{160}{80}\times 820=1640 \text{ turns a mintue.}$$

Example 4. When the armature of a certain machine is making 1200 turns a minute, it can send a current of 11 amperes through an external arc-lamp resistance of 12.5 ohms. The arc lamps are then replaced by 10 incandescent lamps in simple multiple arc, each of them having a resistance of 125 ohms. The resistance of the machine is .05 ohm, and it is required to find at what rate the armature must turn so as to maintain a current of 1.2 ampere through each of these lamps. *Ans.* 1309 turns a minute.

Example 5. A current of 10 amperes is sent through a series of 10 arc lamps, each of 3.8 ohms resistance, by a dynamo machine whose armature is making 1000 revolutions a minute. The arc lamps are then replaced by 8 incandescent lamps, each of 120 ohms resistance and arranged in 4 series of 2 each. The armature is then made to turn at 1044 revolutions a minute. The resistance of the machine is 3 ohms. Assuming that the electro-motive force developed by the machine is proportional to the velocity of rotation, find the strength of current in each lamp.

In the case of the arc lamps the total resistance of the circuit$=3.8\times10+3=41$ ohms, and therefore, by Ohm's law the electro-motive force of the machine is

$$E_1=41\times10=410 \text{ volts.}$$

In the second arrangement the electromotive force which is assumed to be proportional to the velocity of rotation of the armature, is

$$E_2=\tfrac{1044}{1000}\times410=428 \text{ volts,}$$

and the resultant resistance of the circuit is

$$=\frac{2\times120}{4}+3=63 \text{ ohms,}$$

and ∴ the current flowing through the machine

$$=\tfrac{428}{63}=6.8 \text{ amperes;}$$

and the current through each lamp

$$=\frac{6.8}{4}=1.7 \text{ amperes.}$$

Example 6. Fifteen incandescent lamps, each of 120 ohms resistance, are arranged in simple multiple arc between the poles of a dynamo machine whose resistance is 1 ohm, and which is making 1395 revolutions a minute. When the extra polar circuit consists of 3 arc lamps, each of 3 ohms resistance, a velocity of 1550 revolutions develops an electro-motive force of 200 volts. Find the strength of current in each arc lamp, and also in each incandescent lamp.

Ans. Current in arc lamp=20 amperes. Current in incandescent lamp=1.3 ampere.

Example 7. The poles of a certain dynamo machine whose resistance is 0.08 ohm, are in one case connected by 4 incandescent lamps in series, and in another case by the same 4 lamps arranged in multiple arc. The resistance of each lamp is 50 ohms, and the same amount of energy is expended in each case by the driving engine. Compare the amounts of heat developed in the machine.

If H be the total amount of energy developed in the circuit which is the same for both cases, then when the lamps are in series the heat developed in the machine is

$$H_1=\frac{0.08\times H}{200+0.08}=\frac{H}{2501}.$$

In the second case the resistance of the extra polar circuit is$=\frac{50}{4}=12.5$ ohms, and therefore the amount of heat developed in the machine is

$$H_3=\frac{0.08\times H}{12.5+0.08}=\frac{H}{157.25},$$

$$\therefore\frac{H_2}{H_1}=\frac{2501}{157.25}=\frac{16}{1}\text{ nearly.}$$

N.B.—The student will notice from this result that a dynamo machine should never be let run on *short circuit*, as in that case all the energy developed by the machine is dissipated as heat in the machine itself.